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Flow Visualization Experiments on Secondary Flow Patterns in
Curved and Heated Pipes

by



Francis P. Yuen

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES AND RESEARCH
IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE
OF Master of Science

Department of Mechanical Engineering

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THE UNIVERSITY OF ALBERTA
FACULTY OF GRADUATE STUDIES AND RESEARCH

The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research, for acceptance, a thesis entitled Flow Visualization Experiments on Secondary Flow Patterns in Curved and Heated Pipes submitted by Francis P. Yuen in partial fulfilment of the requirements for the degree of Master of Science.

Abstract

The influence of free convection due to centrifugal and buoyancy forces on forced laminar incompressible flow is examined. Flow visualization studies of secondary flow patterns have been carried out at various cross-sections of heated and curved circular pipes using a smoke injection method. Air at room temperature is the fluid medium. Flow is normally directed through a long straight tube to allow a parabolic velocity profile to develop before entering the test section.

In the case of unheated curved pipes, fully developed secondary flow patterns at the exit and downstream of a 180° bend have been investigated. The existence of centrifugal instability in the form of an additional pair of counter-rotating vortices is confirmed. The swirling motion of the developing secondary flow patterns in the entrance region is observed as well. The Dean numbers range from 90 to 827, the Reynolds numbers from 283 to 3911, the angular position between 0° to 180° and the l/d ratio is varied from 0 up to 70.

In isothermally heated horizontal tubes, the inherent ascending developing secondary flow patterns have been obtained for both transient and steady states. Experiments have also been carried out with an isothermally inclined tube for steady state only. The characteristic features of the secondary flow in inclined tubes are clearly delineated. Experiments are conducted for these ranges of parameters:

inverse Graetz numbers from 0.8×10^{-2} to 1.83; Rayleigh numbers (Ra_1) from 3.65×10^4 to 5.18×10^5 ; angle of inclination from 0° up to 60° ; time to heat the tube from 0 to 283 sec.; and constant wall temperatures of 55°C to 95°C .

Secondary flow patterns under the influence of combined effects of buoyancy and centrifugal forces have been studied at the exit of an isothermally heated curved pipe with the bend directed upwards, horizontally and downwards. The Dean numbers range from 22 to 1209 and the products of Reynolds numbers and Rayleigh numbers from 9.49×10^5 to 8.86×10^7 for a range of constant wall temperatures of 55°C to 91°C .

Finally, the secondary flow patterns at the outlet and in the downstream region of a centrifugal blower have also been presented.

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Nomenclature

d	=inside diameter of curved pipe, heated tube or diffuser
d_e	=hydraulic diameter
g	=gravitational acceleration
K	=Dean number, $Re(d/2R_c)^{1/2}$
l	=length of heated tube or diffuser
n	=revolutions per minute, rpm
Pr	=Prandtl number, ν/κ
Ra	=Rayleigh number, $g\beta[T_w-(T_c+T_o)/2]d^3/\nu\kappa$
Ra_1	=Rayleigh number, $g\beta(T_w-T_o)d^3/\nu\kappa$
Ra_2	=Rayleigh number, $g\beta(T_w-T_c)d^3/\nu\kappa$
R_c	=radius of curvature for curved pipe
Re	=Reynolds number, wd/ν
T_c, T_o, T_w	=centreline air temperature at pipe exit, unheated air temperature and average (constant) pipe wall temperature, respectively
T_s	=heating fluid temperature
V_c	=axial centreline velocity
w	=average axial air velocity
x	=distance from exit of bend
z	=dimensionless axial distance from thermal entrance, $(l/d)/RePr$
β	=coefficient of thermal expansion
β_1	=divergence angle of diffuser
θ	=angle of inclination from horizontal direction
κ	=thermal diffusivity

ν	=kinematic viscosity
ϕ	=no. of degrees in angular position from start of bend
τ	=time from start of heating
Γ	= Gr_1/Re^2 where $Gr_1=g\beta(T_w-T_o)d^3/\nu^2$

1. Introduction

1.1 Background Information

Laminar flow in pipes has been one of the most widely studied topics in the fields of engineering heat transfer and fluid mechanics, due to the fact that the secondary motions inside are expected to enhance mixing and momentum transfer between the fluid and its surrounding. Interest in helically coiled tube heat exchangers, compact liquid chromatography columns and aortic arch hemodynamics have led researchers to study the flow in curved pipes. Continuous efforts have been given to the flow in heated tubes due to its importance in many engineering devices such as solar collectors, nuclear reactors, and other applications.

In a circular curved pipe, centrifugal forces are induced due to the presence of the curved surfaces. As the fluid passes through the curved pipes, the faster moving fluid in the central part is driven by a centrifugally induced pressure gradient. By continuity, the slower moving fluid near the wall will flow along the wall to the inner convex wall. Secondary flow results in a pair of counter-rotating vortices in the cross section of the pipe. This vortical motion is then superimposed on the main axial flow and generates a helical path in the fluid motion.

The study of flow in curved pipes is not new. In fact, large number of theoretical and experimental studies have been done covering most aspects of the problem. The

earliest observation of the curvature effect on the flow through a coiled pipe was noticed by Grindley and Gibson[1]' (1908) in an experiment on the viscosity of air. Eustice[2,3] (1910,1911) demonstrated the existence of a secondary motion in a coiled pipe from dye-injection experiments. Dean[4,5] (1927,1928) obtained the first analytical solution for the axial velocity and stream function of the secondary flow. He also found that the flow in curved pipes depends primarily on a new parameter called Dean number $K = Re(d/2R_c)^{1/2}$ (where Re =Reynolds number, d =internal diameter of the circular pipe and R_c =radius of curvature of the pipe), which can be considered as the ratio of the centrifugal force induced by the circular motion to the viscous force. Dean number is a measure of the magnitude of the secondary flow. After Dean's publications, numerous improved theoretical solutions by Barua[6] (1963), McConalogue and Srivastava[7] (1968), and Greenspan[8] (1973) have been reported. The most recent works include Van Dyke[9], Dennis[10,11] and Masliyah[12]. More information can be found from a detailed survey conducted by Nandakumar and Masliyah[13], and Berger and Talbot[14].

In an isothermally heated tube, the presence of density difference as the fluid particles being heated introduces significant buoyancy forces in the thermal entrance region of the flow field. As the fluid enters the heated tube, a boundary layer develops on the wall with secondary flow

'numbers in square brackets designate Reference at the end of each chapter.

developed inside the boundary layer by the buoyancy forces due to the temperature difference between the wall and the inlet fluid. The central fluid will be accelerated by the displacement of the axial boundary layer which causes the fluid particles to move toward the centre of the pipe. As well, a downward stream in the cross section of the pipe is also induced by the upward displacement of the secondary boundary layer. The combination of the downward stream with the radial stream results in the formation of two vortices in the central core of the pipe.

The problem of flow in an isothermally heated pipe was first solved analytically by Graetz[15] (1883) and later by Nusselt[16] (1910) with fully developed laminar flow. Further investigations have been done by Drew[17], Colburn[18], and many others and the earlier results have been summarized by McAdam[19]. The problem was then later studied by Cheng and Ou[20,21], Yao[22,23], Yousef and Tarasuk[24], and Hishida et al[25]. For the uniform wall temperature boundary condition, the bulk fluid temperature approaches the wall temperature in the fully developed forced convection region as the asymptotic Nusselt number approaches $Nu_{\infty}=3.66$.

Flow in a uniformly curved pipe and in an isothermally heated or cooled tube is considerably more complex than that in straight conduits because of the additional induced body forces. Among all the available experimental results, the measurements of axial pressure gradients, shear stresses and

friction factors have received the most attention. While there is an abundance of literature on theoretical and experimental studies on flow in curved pipes and in heated tubes, in contrast little work is done on this subject by the flow visualization approach. As described by Cheng in [26], flow visualization studies have been carried out extensively by many investigators since the pioneer work of Reynolds[27] (1883) who injected dye into water flowing inside a horizontal tube for his experiment on transition from laminar to turbulent flow. However, it is noted that literature of flow visualization studies on secondary flow patterns in curved and heated pipes are rather limited. It is believed that some good flow visualization results will enable us to further understand the secondary flow structures and possibly complement some of the theoretical predictions.

1.2 Scope of Study

This thesis will focus on secondary flows caused by the centrifugal force effects in curved pipes; and by the buoyancy force effects in the thermal entrance region of isothermally heated tubes. The problem will be approached by means of flow visualization technique. Our main emphasis will be on secondary flow patterns at cross-sections of circular pipes or tubes with laminar incompressible viscous flow. The different conditions examined are: fully developed or developing flow, parabolic or uniform entrance

velocity profile and steady or transient state for the cases of unheated curved pipes, isothermally heated horizontal or inclined tubes, and isothermally heated curved pipes.

In this thesis, flow visualization is made possible by smoke injection methods. This visualization method was first employed by Cheng et al[28] in 1979, where cigarette smoke was used. An intensive study of various flow visualization methods were undertaken by Mok[29] in 1983. The smoke injection method by which hydrochloric acid was injected into air with ammonia vapor was found to be the most effective way to study the secondary flow patterns in unheated curved pipes. The same smoke generating technique was employed initially, but was found to be inappropriate in the experiments with isothermally heated tubes and entry flow in curved tubes. A new smoke generating technique was then applied during this work to cope with different situations.

The first objective is to investigate the centrifugal induced secondary flow in curved tubes. A fully developed flow case has been examined by observing the secondary flow patterns at the exit of a 180° bend. Moreover, the existence of the centrifugal instability phenomenon is to be confirmed. Finally, the decay of such centrifugal instability in a straight section downstream of the bend is examined as well (Chapter 2).

The developing secondary flow patterns in the entrance region of an unheated curved tube are presented (Chapter 3).

The second objective is to investigate the buoyancy induced secondary flow in heated straight pipes. The developing secondary flow patterns in the thermal entrance region of isothermally heated horizontal pipes have been observed (Chapter 4).

Only Chapter 5 is based on transient state flow. Transient secondary flow patterns in a heated horizontal pipe have been investigated.

One may wonder how the buoyancy induced secondary flow patterns change in an inclined case as the body force is subject to an angle of inclination. The development of secondary flow patterns in the thermal entrance region of an isothermally heated inclined pipe have been examined (Chapter 6).

The third objective is to observe the interaction between the centrifugal and buoyancy forces on the secondary flow patterns at the exit of a heated curved pipe (Chapter 7).

A final attempt is performed to obtain the secondary flow patterns under the influence of centrifugal and Coriolis forces at the outlet and in the downstream region of a centrifugal blower at low fan speed (Chapter 8).

1.3 Format of Presentation

In view of different aspects of the investigation undertaken, it was decided that each chapter would be written in accordance with each individual study. While a

paper-format is being used, each chapter will consist of a summary, an introduction, an experimental apparatus and procedure section, its results, conclusions and its own references. In certain chapters, details in the introduction and experimental apparatus and procedure sections will be excluded to avoid repetition. A final chapter will then link the separate findings together, providing an overall result of the study.

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2. Secondary Flow at the Exit and Downstream of a 180 Degree Curved Tube²

Summary

Photographs are obtained for secondary flow patterns in straight tubes downstream ($x/d=0-70$) of a 180° bend (tube inside diameter $d=2.54$ cm, radius of curvature $R_c=12.7$ cm). Air at room temperature has been used as the flowing fluid. Each test section is preceded by a long entrance length. For curved pipes, the Dean number range is $K=90$ to 572, and Reynolds number range is $Re=283$ to 1807. At the exit of 180° bend, the onset of centrifugal instability in the form of an additional pair of Dean vortices near the central outer wall was found to occur at Dean number of about $K=100$.

²A similar version of this chapter has been published. Cheng, K.C. and Yuen, F.P. ASME paper 84-HT-62. 22nd ASME/AIChE National Heat Transfer Conf., Niagara Falls, New York, Aug. 5-8, 1984.

2.1 Introduction

The secondary flows caused by the centrifugal force in curved pipes are known to be important in laminar forced convective heat transfer. The literature related to this area of study is quite extensive because of their importance in heat transfer equipment applications.

After the classical work of Dean[1,2] on fully developed laminar flow in curved pipes, many theoretical and experimental investigations on flow and forced convective heat transfer in curved pipes have been carried out and the literature has been well surveyed recently by Berger, Talbot and Yao[3], and Masliyah and Nandakumar[4]. One also notes that heating and cooling coils are commonly used in heat transfer equipment. The physiological applications of curved tubes dealing with flow patterns and wall shear stress in arteries are given by Pedley[5].

The numerical solutions on fully developed laminar flow in curved pipes or ducts, usually yield secondary flow patterns. For laminar flow in a curved pipe, it is generally understood that the secondary flow consists of a pair of symmetric counter-rotating vortices in a cross-section normal to the main flow. However, it is not clear whether the secondary flow eventually decays or changes into other flow patterns as the Reynolds number increases to the turbulent flow regime. For flow in curved pipes, it is observed that the pressure increases monotonically from the inner convex curved wall toward the

outer concave curved wall across the cross-section due to the centrifugal forces caused by the curvature effect. However, the centrifugal force increases and then decreases toward the outer concave wall after reaching a maximum value along a line parallel to the horizontal axis. Thus, the region near the concave outer wall is potentially unstable and centrifugal instability (Dean's instability) phenomenon[6] may occur. Dean's instability phenomenon in the form of an additional pair of vortices occurring near the centre of the concave outer wall has been observed by flow visualization for fully developed laminar flow in curved square channels at a certain Dean number[7]. It is thus speculated that centrifugal instability phenomenon may occur for other geometrical shapes such as curved circular or semicircular pipes. Indeed recent numerical solutions for fully developed laminar flow in curved pipes reveal the occurrence of the above noted centrifugal instability for curved semicircular tubes[8] and curved circular tubes[9,10]. Recently the flow visualization studies on secondary flow patterns and centrifugal instability in curved circular and semicircular pipes have been reported in [11]. It is noted that literature of flow visualization studies on secondary flow patterns in a 90° or 180° bend and downstream are rather limited. The local heat transfer measurements in a U-bend and downstream has been reported by Moshfeghian and Bell[12] and the pertinent literature can be found. The secondary effect of a 90° bend and downstream in

a straight tube has also been studied experimentally by several investigators. It is evident that theoretical and experimental studies on flow in or downstream of a 90° or 180° bend should be complemented by flow visualization studies, particularly on developing or decaying secondary flow patterns across the cross-section normal to the main flow.

The objective of this chapter is to investigate the secondary flow patterns at the exit of a 180° bend and downstream region in a straight tube for a range of Reynolds numbers $Re=283-1807$. The flow visualization was facilitated by injecting smoke into air. The centrifugal instability phenomenon (Dean's instability) for laminar flow in curved circular pipes is clearly demonstrated by these flow visualization studies.

2.2 Experimental Parameters

The experimental parameters for steady viscous flow in curved circular pipes are curvature ratio (ratio of tube inside diameter to coil diameter) $d/2R_c$, Reynolds number Re and Dean number. For this experiment, $d=2.54\text{cm}$, $R_c=12.7\text{cm}$, $Re=283-1807$, and $K=90-572$.

2.3 Experimental Apparatus

The experimental apparatus for a 180° bend with a downstream straight tube is shown schematically in Fig.

2.1³. The 180° bend was machined from two pieces of plexiglas and joined together at the plane of symmetry. The downstream straight section utilizes plexiglas tubes with different lengths. Compressed laboratory air was used as the flow medium. A Fulflo filter was employed to remove any undesirable debris and moisture from the air. The air was then passed to a main control valve which controls the flow rate. The flow rate of air was measured by a Meriam laminar flow meter (model 50MW20-1).

After the laminar flow element, the air was split into two separate lines, both of which led to a settling chamber. One of these lines passed air through a jar of ammonium hydroxide. The other line bypassed the ammonium hydroxide jar. By adjusting the flow rate of these lines, concentration of ammonia carried over can be adjusted. Ammonia hydroxide in the air stream will provide a means of flow visualization.

The settling chamber consists of a steel drums with several sections of screen in order to provide a uniform air flow. It is also streamline-shaped at the end to smooth the flow. The 180° bend was preceded by a 4.7m long entrance length, the length of which was determined to ensure the flow at the inlet of the bend is fully developed.

³first number denotes Chapter. Figures are grouped at the end of each chapter in order they referenced to.

2.4 Experimental Procedure

2.4.1 Measurement of Air Flow Rate

A Merian laminar flow element was used to measure the air flow rate. This flow element consists of numerous longitudinal triangular passages to ensure flow is laminar. The linear relationship between the friction loss and the flow rate in the laminar region is calibrated by the manufacturer. The differential pressure across the element was measured by a Dwyer Airbar inclined manometer. The flow rate was then obtained from a calibration curve provided by the manufacturer (See Fig. 2.2). Overall calibration error was no more than 0.1%.

2.4.2 Visualization Method

Flow visualization was facilitated by a smoke injection method. The air supply, after being passed through the jar of ammonia hydroxide, carried vapor of ammonia into the test section. The amount of the vapor was small compared to the air and therefore did not affect the flow measurements. Hydrochloric acid was then introduced through a small hole in front of the bend using a hypodermic needle. Hydrochloric acid and the ammonia vapour reacted to form ammonium chloride (NH_4Cl) in the form of very fine white powder. This powder was so fine and was dispersed into the streamlines. These streamlines showed distinctively the flow patterns at the curved test section.

2.4.3 Photograph Observation

The secondary flow patterns, which were photographed from a direction facing the exit, were observed at the exit of the 180° bend ($x=0$) and at various downstream sections at a distance x from the outlet of the bend. Illumination was provided by a thin plane of light from a 500W projector. The plane of light was obtained by putting two razors close together in a slide mounting frame. Pictures were taken in a dark surroundings where only the smoke-filled flow in a plane transverse to the pipe axis was highlighted by the illumination. The air at the curved pipe exit was discharged into the atmosphere. A Nikon FM2 single lens reflex camera with a 50mm micro lens was employed. Kodak Tri-X black and white film was used throughout the tests. The aperture used was $f3.5$ while the shutter speed ranged from $1/2$ - $1/8$ sec. For most cases, a speed of $1/15$ sec. gave satisfactory pictures.

2.5 Results and Discussion

2.5.1 Secondary Flow Patterns at the Exit of 180° Bend

(Curvature Ratio $d/2R_c=0.10$)

In view of the onset of centrifugal instability phenomenon for fully developed laminar flow in a curved pipe reported by recent numerical solutions[9,10], the experimental confirmation of this phenomenon is of particular interest. It is also good to note that three

types of secondary flow patterns for fully developed laminar flow in curved pipes have been reported in the existing literature corresponding to small Dean number ($K \leq 20$), intermediate Dean number ($20 \leq K \leq 200$) and large Dean number ($K \geq 200$). The secondary streamline patterns for small Dean numbers were identified by Dean's analysis[1,2] based on perturbation solution. The secondary streamline patterns for the intermediate range of Dean numbers were identified by various numerical solutions in recent years up to about $K=400$ [10]. The secondary streamline patterns for large Dean number range is a physical model based on potential core and boundary layer approximation near the pipe wall. The core-boundary layer model enabled several investigators to obtain theoretical solutions for large Dean number flow regime[13]. Apparently the onset of centrifugal instability posed some questions regarding the validity of the physical models used for large Dean number flow. Thus the precise details of the secondary flow patterns for large Dean number flow regime up to turbulent flow should be studied by both numerical methods and flow visualization. In this connection, it is proposed that one or more pairs of the additional counter-rotating vortices observed at a certain Dean number in curved rectangular channels with various aspect ratios[7,14], curved semicircular tubes[8,11], curved circular tubes[9,10] and various other geometrical shapes to be identified in future investigations be called "Dean vortices". It is interesting to observe that Taylor,

Gortler and Dean vortices are all longitudinal vortices with the same nature and origin.

The change of secondary streamline pattern at the exit of 180° bend with the increase of Dean number K or Reynolds number Re is shown in Figs. 2.3 and 2.4 for two series of experiments. The pictures on the right-hand side were obtained by disturbing the flow using a hypodermic needle at 90° position from the start of the 180° bend, whereas those on the left-hand side represent no disturbance. At Dean number less than $K=90$, the secondary flow patterns are quite familiar comprising a pair of symmetric vortices. However, one notes the disturbance near the centre of the outer concave wall. At $K=99$, an additional pair of counter-rotating vortices (Dean vortices) appears and at $K=184$, one can clearly see the formation of the Dean vortices. The numerical solutions by other investigators reveal that a four-vortex solution already appears at $K=113.3$ [9] and $K=114.2$ [10]. The agreement between numerical solution and experiment is reasonable and one may conclude that the critical Dean number for the onset of centrifugal instability is about $K=100$. The development of secondary flow patterns with the increase of Dean number and the difference between the natural and forced disturbances are of special interest. At $K=572$, the secondary flow pattern becomes quite complicated. The existence of an additional pair of vortices near the outer wall of the cross-section in a 180° bend was noted by Rowe[15] at high Reynolds number

$Re=2.36 \times 10^5$ ($K=4.72 \times 10^4$). However his secondary flow patterns were inferred from both the yaw measurements and the shape of the pressure contours.

Fig. 2.5 shows a comparison of some experimental results (right-hand side of each photograph) from this study with the ones obtained by others using numerical solutions (left-hand side). Figs. 2.5(a) and (c) consist of a pair of symmetrical vortices at $K \approx 200$ and $K \approx 370$ respectively, while Figs. 2.5(b) and (d) show their four-vortex solutions consisting of two symmetrical vortex pairs. A fairly good agreement between numerical and experimental solutions is seen, except some discrepancies on the flow pattern at the region above the tube centre where the flow is potentially unstable.

2.5.2 Secondary Flow Patterns in a Straight Section

Downstream of the 180° Bend ($d/2R_c=0.10$)

It is expected that the intensity of secondary flow after a 180° bend will gradually decay and eventually revert to the Poiseuille profile for fully developed laminar flow in a straight pipe. Rowe[15] shows the estimated secondary flow patterns at 1, 5, 29 and 61 pipe diameters in a straight section downstream of a 180° bend at Reynolds number of 2.36×10^5 . The gradual decay of secondary flow patterns is shown in Figs. 2.6, 2.7 and 2.8 for Dean numbers $K=127$, 208 and 312 respectively. Here, an attempt is made to present more pictures of secondary flow patterns at

various downstream positions for future reference. This is because the secondary streamlines of the type shown in Figs. 2.6 to 2.8 do not seem to be available from either experiment or numerical solution in the literature. It is noted that the pictures on the right-hand side were again obtained by disturbing the flow using a hypodermic needle at 90° position from the start of the 180° bend. And those on the left-hand side represent natural disturbances. All the results show that the secondary flow persists at a downstream position $x=70d$ although the intensity of secondary flow may be weak there.

At $K=127$, the change of the shape for Dean vortices are clearly seen and the interaction between the Dean vortices and the original primary vortices as a function of downstream distance x is of interest. At $x=30d$, the Dean vortices seem to be engulfed by the original secondary vortices. At $x \geq 50d$, the secondary flow patterns appear to be unstable. The secondary flow patterns in the downstream section become more complicated at $K=208$ since yet another additional pair of vortices appears to form below the Dean vortices at $x=1d$. These vortices persist even at $x=40d$. However, the appearance of the Dean vortices is not clear except the boundary of its territory. At $K=312$, a small disturbance by the hypodermic needle at 90° position from the start of the 180° bend was found to have a considerable effect on secondary flow patterns in the downstream straight section. In this case, the boundary of the Dean vortices

becomes vague at $x \geq 15d$.

2.6 Concluding Remarks

The secondary flow patterns at the exit of a 180° bend have been investigated using the smoke injection method. Photographs are shown for Dean number range of $K=90$ to 572 .

The presented photograph results confirm the existence of the centrifugal instability in the form of the occurrence of the Dean vortices at the exit and downstream of the 180° bend. The onset of centrifugal instability was found to occur when Dean number is about $K=100$.

For secondary flow patterns in a downstream straight section after the 180° bend, the experimental range for dimensionless axial distance is $x/d=1$ to 70 for Dean numbers $K=127, 208$ and 312 .

These secondary flow patterns are useful for future comparison with predictions from numerical and analytical solutions.

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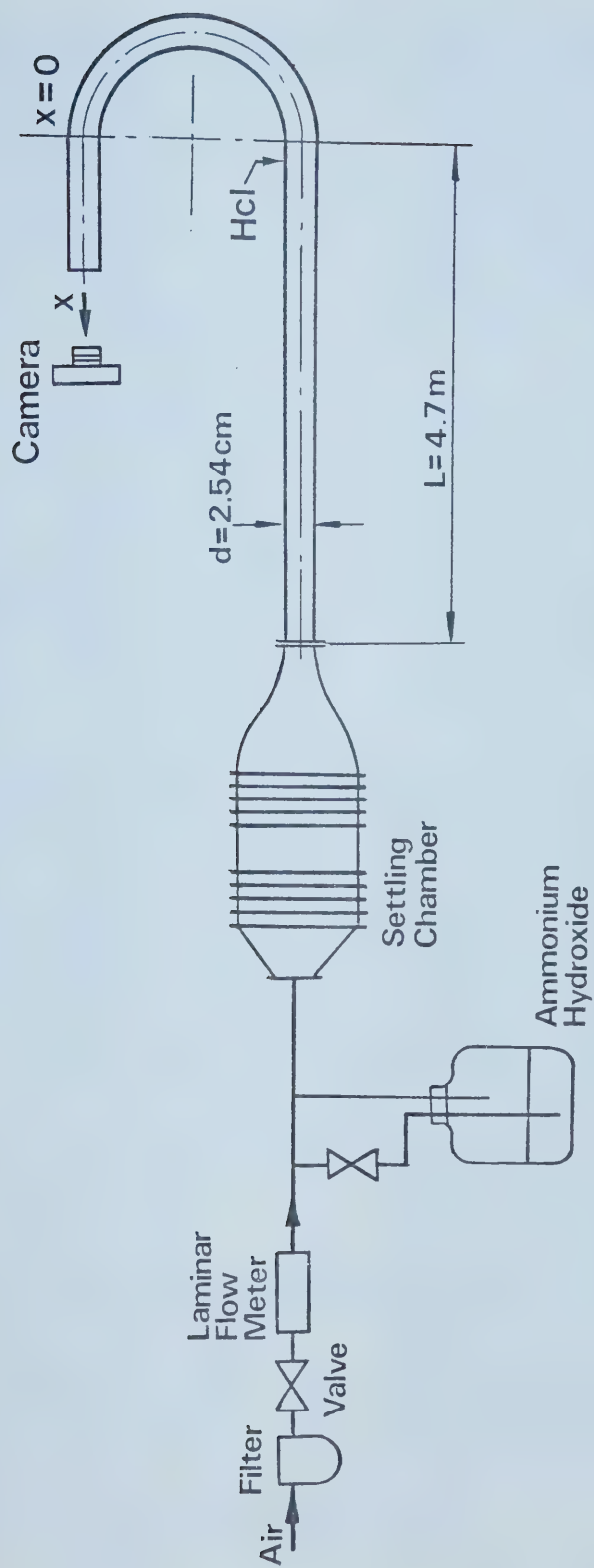
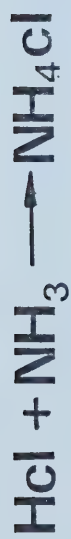


Fig. 2.1 Schematic diagram of experimental apparatus.

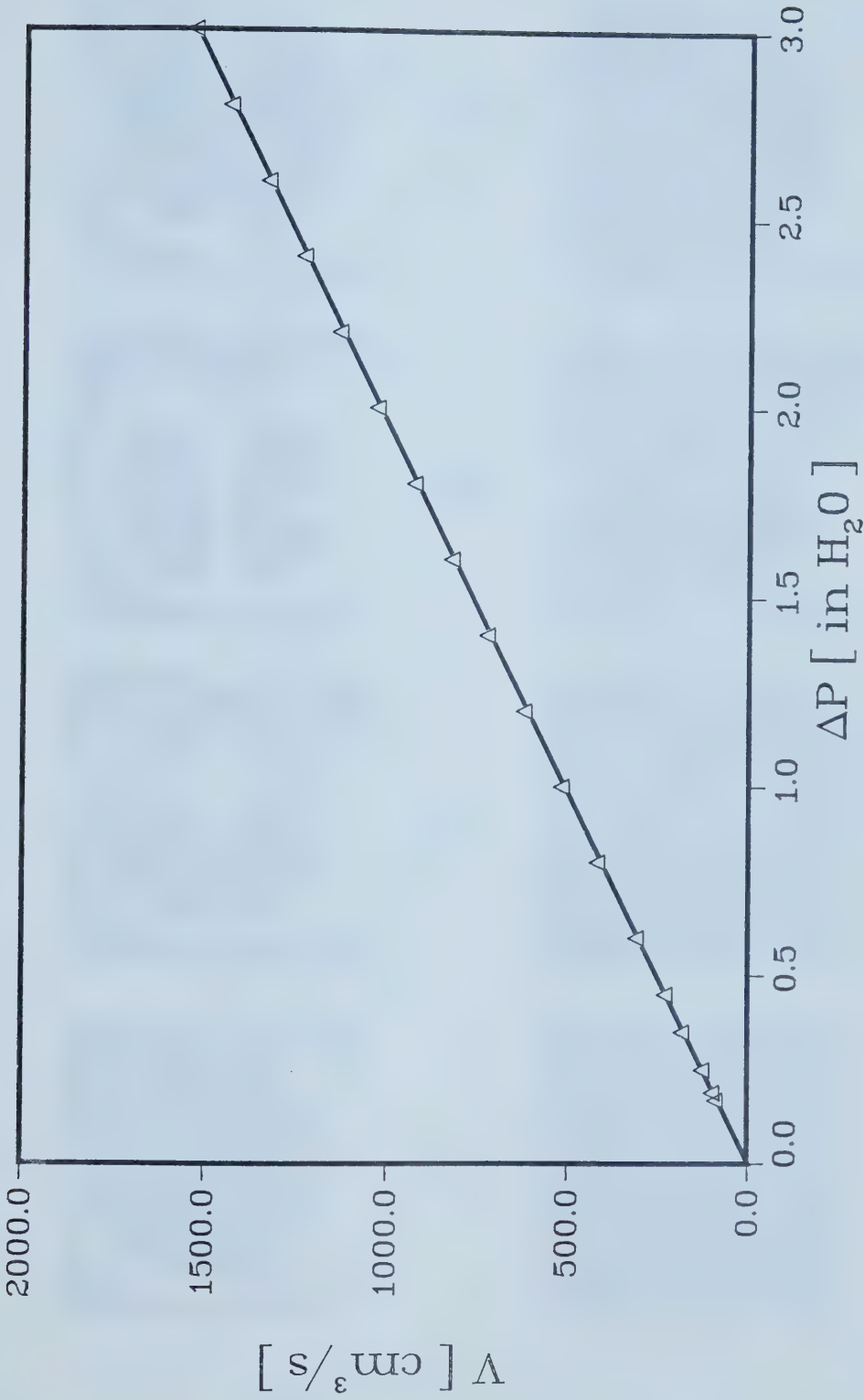


Fig. 2.2 Calibration curve for the laminar flow element (Model 50MW20-1) from Meriam.

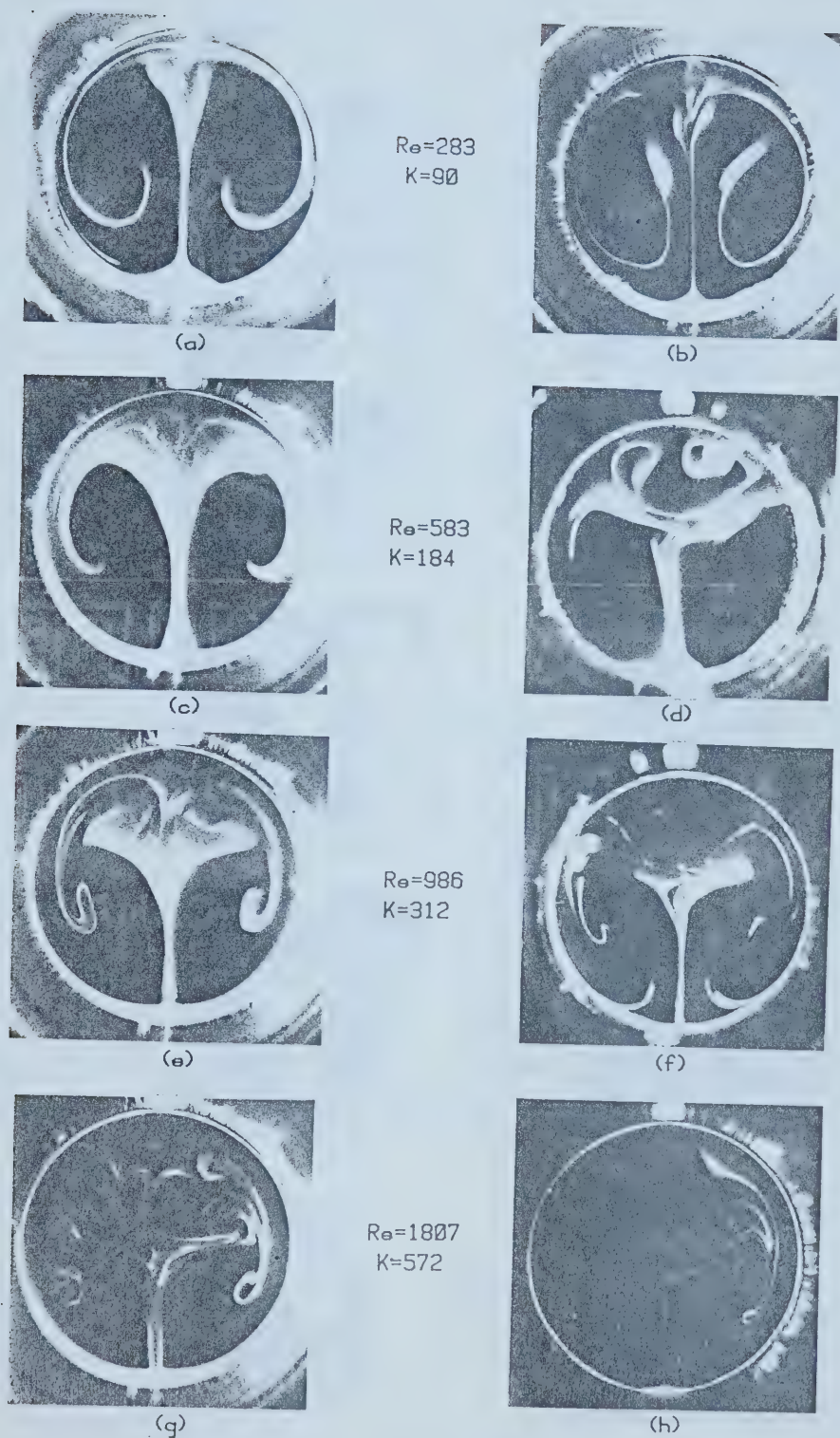


Fig. 2.3 Secondary flow patterns at the exit of a 180° bend.

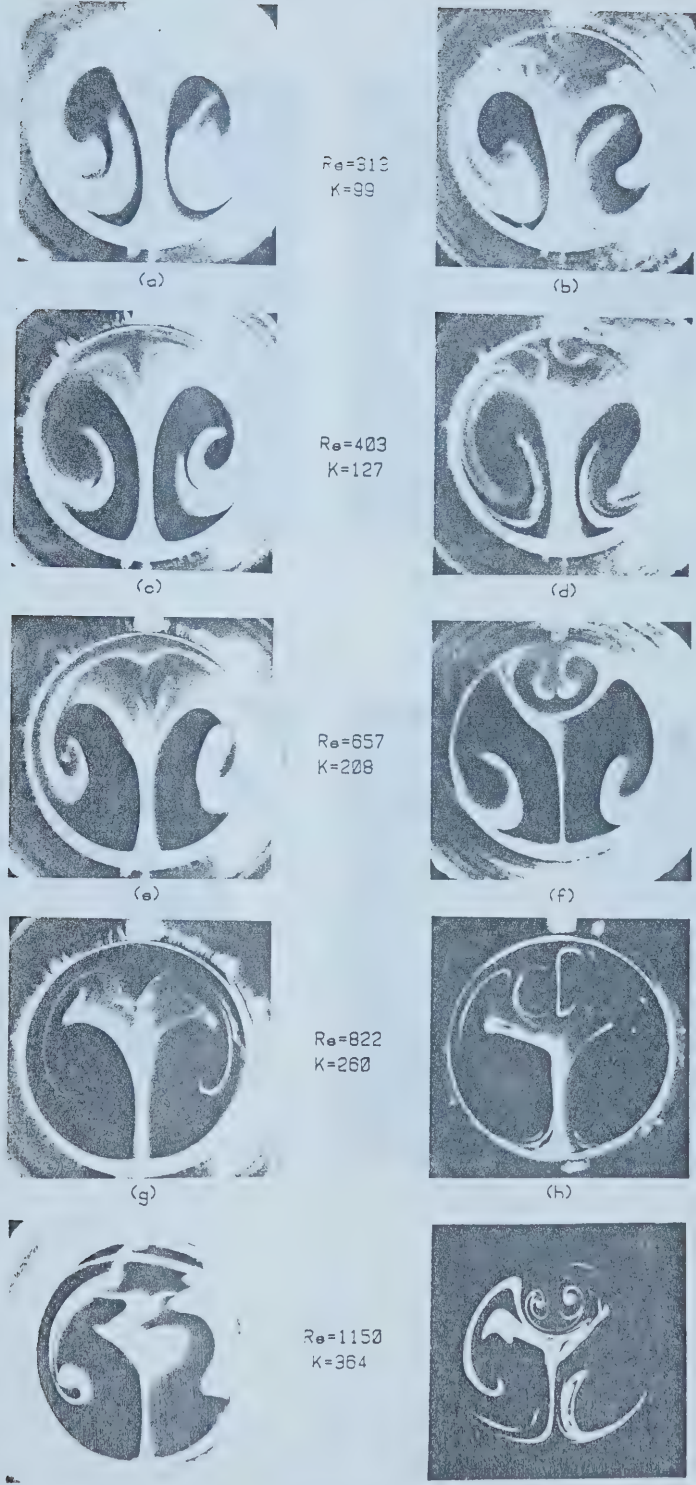
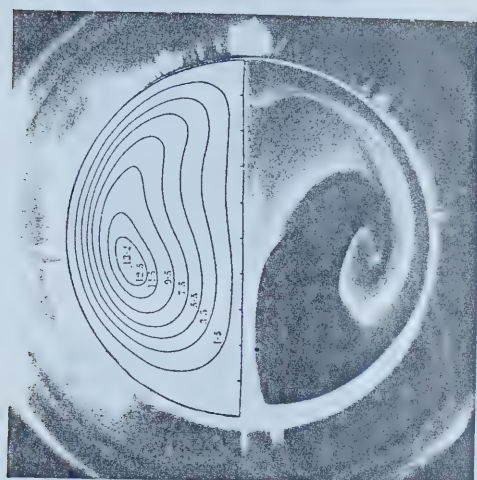


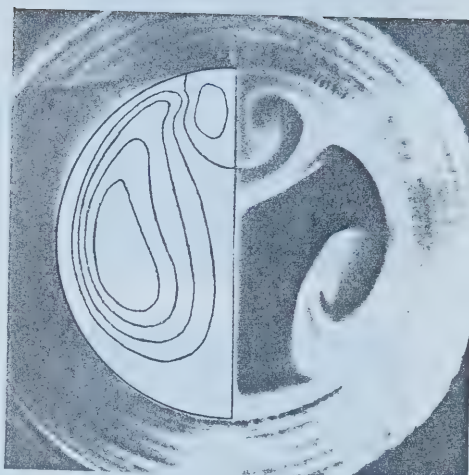
Fig. 2.4 Secondary flow patterns at the exit of a 180° bend.



K=191

K=208

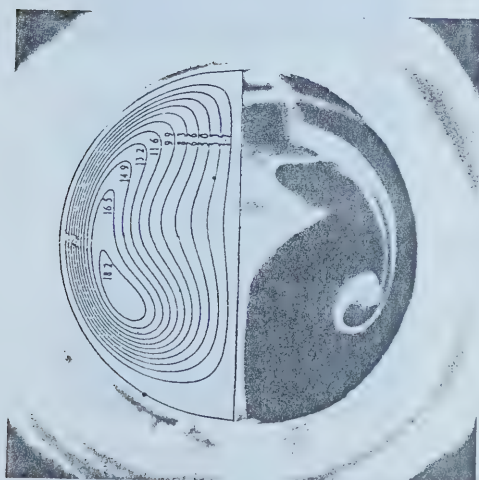
(a)



K=202

K=208

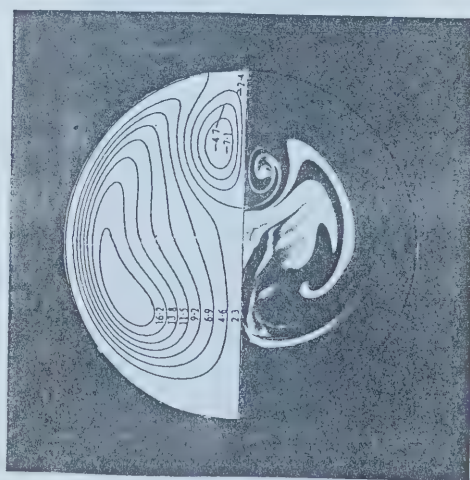
(b)



K=374

K=364

(c)



K=374

K=364

(d)

Fig. 2.5 Comparison of some present work with other investigators. Contours of stream function flows on the left-hand side of each figure were taken from:
 (a) Collins and Dennis[16]
 (b) Nandakumar and Masliyah[9]
 (c) & (d) Dennis and Ng[10].



Fig. 2.6 Secondary flow patterns in a straight section downstream of a 180° bend at $Re=403$, $K=127$.



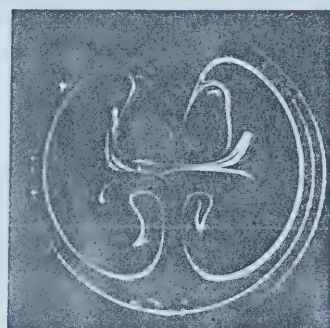
(i)

 $X=10d$ 

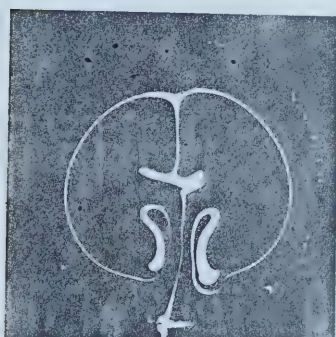
(j)



(k)

 $X=15d$ 

(l)



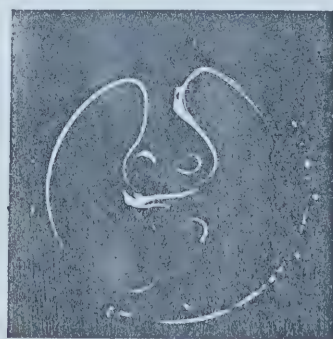
(m)

 $X=20d$ 

(n)



(o)

 $X=30d$ 

(p)

Fig. 2.6 Continued.

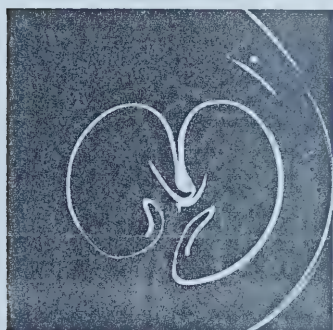


(q)

X=40d



(r)

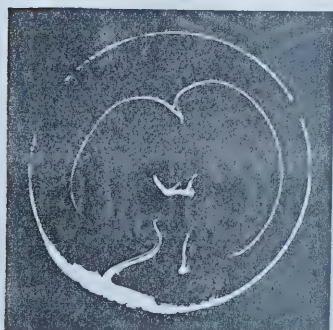


(s)

X=50d



(t)

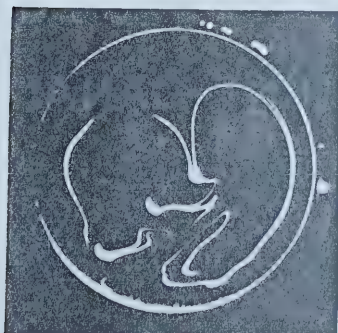


(u)

X=60d



(v)



(w)

X=70d

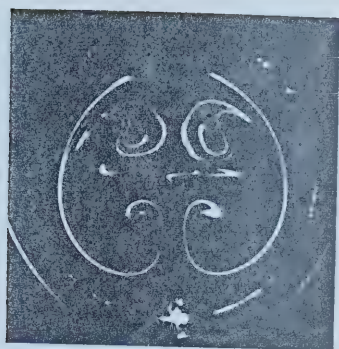


(x)

Fig. 2.6 Continued.



Fig. 2.7 Secondary flow patterns in a straight section downstream of a 180° bend at $Re=657$, $K=208$.

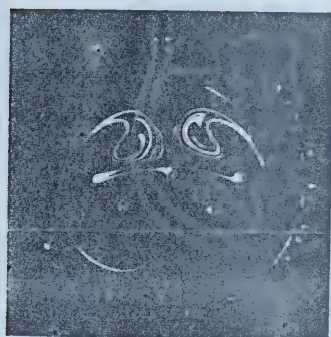


(i)

X=10d

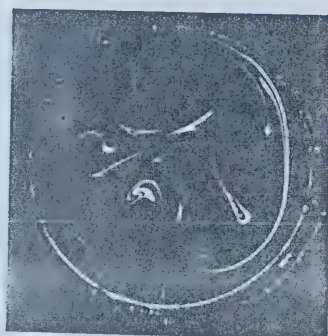


(j)

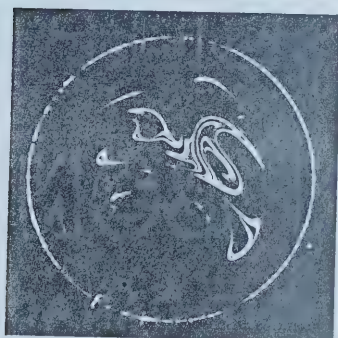


(k)

X=15d

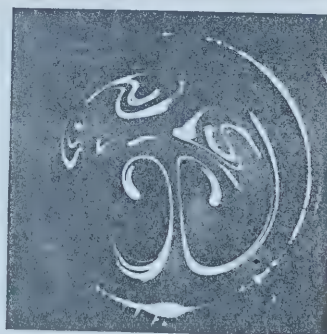


(l)

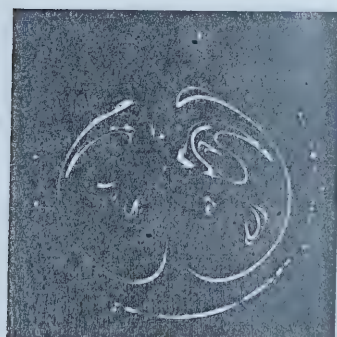


(m)

X=20d

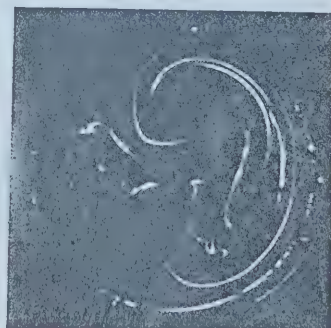


(n)



(o)

X=30d



(p)

Fig. 2.7 Continued.

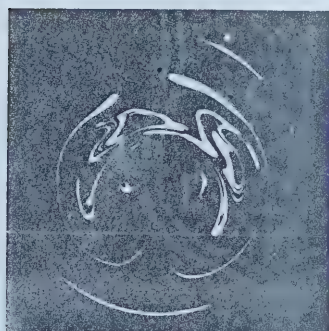


(q)

X=40d



(r)



(s)

X=50d



(t)



(u)

X=60d



(v)



(w)

X=70d



(x)

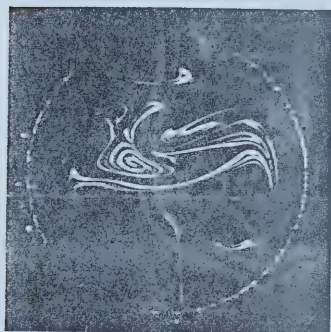
Fig. 2.7 Continued.



(a)

 $X=1d$ 

(b)



(c)

 $X=3d$ 

(d)



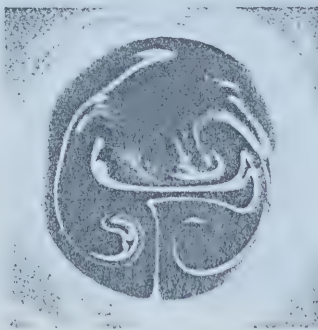
(e)

 $X=5d$ 

(f)



(g)

 $X=7d$ 

(h)

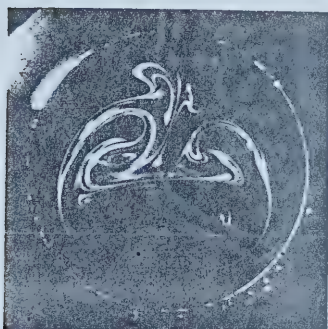
Fig. 2.8 Secondary flow patterns in a straight section downstream of a 180° bend at $Re=986$, $K=312$.



(i)

 $X=10d$ 

(j)



(k)

 $X=15d$ 

(l)



(m)

 $X=20d$ 

(n)

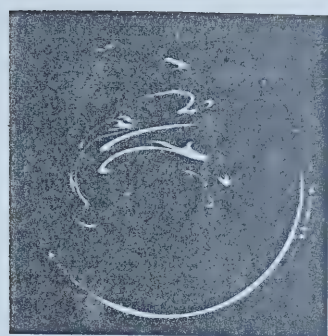


(o)

 $X=30d$ 

(p)

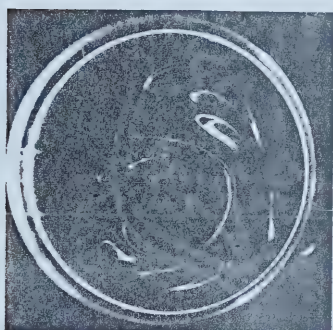
Fig. 2.8 Continued.



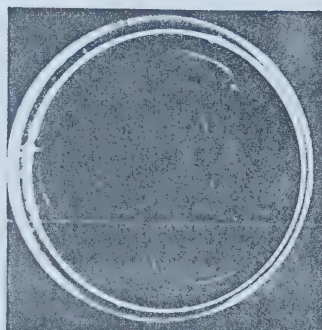
(q)

 $X=40d$ 

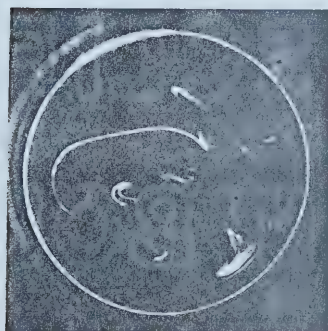
(r)



(s)

 $X=50d$ 

(t)



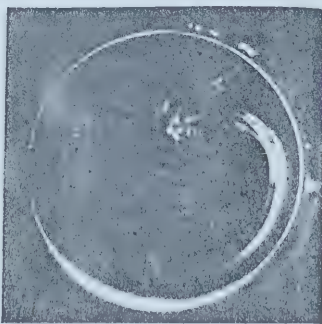
(u)

 $X=60d$ 

(v)



(w)

 $X=70d$ 

(x)

Fig. 2.8 Continued.

3. Developing Secondary Flow in Curved Tubes

Summary

Flow visualization was carried out to study the development of steady viscous flow in the entry region of curved circular tubes with a circular ratio of 0.10. Both parabolic and uniform entrance flows were included covering the Dean number range of $K=90-1244$. Air was the fluid medium. Photographs were taken at five different sections of the entry regions. These section were equally spaced along the circumference at 0° , 45° , 90° , 135° and 180° from the start of bend. Visualization was made possible by injecting smoke into air. The secondary flow patterns shown are in qualitative agreement with numerical and experimental solutions by recent researchers, and reveal the complexity of this problem.

3.1 Introduction

Flow in curved pipes has been one of the most fundamental interests in fluid mechanics due to its importance in biomechanic and engineering applications. After the classical work of Dean[1,2] in 1927 and 1928 on fully developed laminar flow in curved pipes, many theoretical and experimental investigations on flow and forced convective heat transfer in curved pipes have been carried out and the literature is well surveyed by Berger, Talbot and Yao[3], and Masliyah and Nandakumar[4].

The flow development in the entry region of a curved pipe is generally understood as follows[5]: As the fluid enters a curved pipe, the flow field consists of two regions. A thin boundary layer is developed near the wall where the viscous forces are balanced by the inertia forces and the centrifugal force has a second order effect. An inviscid core in which the centrifugal force due to curvature is balanced by a pressure gradient force directed toward the centre of curvature. In the boundary layer, a transverse flow which is induced by the inward pressure force on the core moves from the outer bend toward the inner bend. As the flow develops further downstream, the curvature effects become as important as the viscous and inertia effects in the boundary layer. The growth of the boundary layer accelerates the flow in the core, while the transverse flow in the boundary layer then induces a cross-flow from the inner to the outer bend. Thus, a

secondary flow is induced by the curvature with the slower-moving fluid in the boundary layer on the wall moving inward, and faster-moving fluid in the core moving outward. The secondary boundary layer thus acts as a sink to receive the fluid near the outer wall and as a source of fluid near the inner wall.

While the studies on fully developed steady flow through curved tubes have been fairly extensive, only recently has entry flow in curved pipes attracted some attention. Theoretical investigations in this area can be found in [5-12] and experimental studies in [13-17]. An up-to-date literature review can be found in Soh's PhD thesis[11] and will not be repeated here. As mentioned by Argawal et al[14], the quantitative agreement between theory and experiment for entry flow in curved pipes is poor. In Akiyama's work[16] (1980), the developing secondary flow in the entry region of a curved pipe is shown by means of vector indicators only. More careful work has to be done before this problem can be clarified. In view of the available experimental results, it is noted that flow visualization studies on the development of secondary flow patterns in the hydrodynamic entrance region of curved pipes are rather limited. In the entrance region of curved pipes, it is of particular interest to see the length of tube required to allow for complete flow development (fully developed), the development of the secondary flow pattern and the formation of secondary flow separation.

The purpose of this chapter is to investigate the developing secondary flow patterns in the hydrodynamic entrance region of curved pipes. Photographs were taken at the exit of bends ranging from 0° to 180° with both uniform and parabolic entrance flows. The flow visualization was made possible by injecting smoke into the air using two different methods. The secondary flow patterns are presented to illustrate the centrifugal force effect in the hydrodynamic entrance region of curved pipes. The results can be useful for future comparison with numerical solutions and confirming the theoretical model.

3.2 Experimental Parameters

The experimental parameters for steady viscous flow in curved circular pipes are curvature ratio $d/2R_c$, Reynolds number $Re = wd/\nu$, Dean number $K = Re(d/2R_c)^{1/2}$ and circumferential increments ϕ . In this experiment, $d = 2.54\text{cm}$, $R_c = 12.7\text{cm}$, $Re = 283\text{--}3911$, $K = 90\text{--}827$ and $\phi = 0^\circ\text{--}180^\circ$.

3.3 Experimental Apparatus and Procedure

A schematic diagram of the experimental apparatus is shown in Fig. 3.1. Compressed room air was used as the fluid medium. A Fulflo filter was employed to remove any undesirable debris and moisture from the compressed air. The flow rate of air was measured by a Meriam laminar flow meter which has a passage consisting of numerous triangular sections. To determine the flow rate, the pressure

difference between the locations of the flow meters was measured by a Dwyer Airbar inclined manometer. The pressure difference was then used on the calibration curve provided by the flow meter manufacturer to obtain the flow rate. The calibration error of the laminar flow meter should be less than 0.3% and the visual error of reading the inclined manometer is approximately 1.7%.

The main air supply, after being passed through the filter, pressure regulator and flow meter, was split into two separate lines. One of the lines was first connected to the smoke generating device before both lines joined together and entered the settling chamber. In this experiment, smoke was generated using two different methods. For the parabolic flow development test, air was passed into a jar of ammonia hydroxide while hydrochloric acid was injected through a hypodermic needle ahead of bends. For the uniform flow development test, smoke was generated by burning straw paper sticks inside a tube. The amount of ammonia vapor or smoke was relatively small compared to the air so that the effect of the ammonia vapor on the fluid properties is negligible.

The settling chamber, which consists of several layers of screens, was used to divert the flow so as to obtain a uniform air flow. For the uniform flow test, a very slightly tapered spacer (approximately 0.95cm thick) was required between the settling chamber and the bends. For the parabolic flow test, an entrance length of 4.7m was used

to ensure fully developed flow at the inlet of the bends.

Four different sections of bends, namely 45° , 90° , 135° and 180° , were machined in two halves from plexiglas and joined together at the plane of symmetry. The secondary flow patterns were observed at the cross-sections 1.3cm away from the exit of bends. A slit light source was provided by a 500W slide projector. A Nikon FM2 single lens reflex camera with a 50mm micro lens was used with Kodak Tri-X black and white film and photographic settings of f3.5 and $1/4-1/8$ sec.

3.4 Results and Discussion

3.4.1 Secondary Flow Development at Bends with Parabolic

Entrance Velocity Profile(Curvature ratio $d/2R_c=0.10$)

The secondary flow development in the entrance region of a curved pipe at certain Dean numbers $K=90-727$ are shown in Fig. 3.2 with no disturbances. The top of each photograph represents the outside of bend and the bottom represents the inside of bend. Since there is no provision to prevent illumination light from shining on the upstream region of bend, double images or different colored regions have occurred throughout the photographs obtained. It is therefore suggested to focus on the darker region which represents the actual flow pattern occurring in the region being studied.

An empirical correlation expressing the development was given by Austin[7] as:

$$D = 49.0 (N)^{1/3}$$

where, $N=Kd/2Rc$

D =No. of degs. req'd for development.

Based on the above correlation, Table 3.1 shows the degrees required for fully developed flow at different Dean numbers used in this experiment:

Table 3.1 Degrees required for development

K	94	208	364	520	727	827
D, deg.	102	135	162	183	205	213

In comparing the above values to Fig. 3.2, one may conclude that all flow patterns in Fig. 3.2 are still in a developing stage, except flow patterns at $\phi=135^\circ$ in Figs. 3.2(a) through 3.2(d) and at $\phi=180^\circ$ in Figs. 3.2(a) through 3.2(e) which are in fully developed. Fig. 3.2(a) does not seem to be in agreement with Austin's correlation as the secondary flow patterns at $\phi=135^\circ$ for $K=90^\circ$ are still in developing stages. Unfortunately flow patterns at $\phi=135^\circ$ in Figs. 3.2(b) through 3.2(b) are not very clear and it is difficult

to compare them with the numerical solutions.

Generally speaking, our results agree quite well with Austin's experiments regarding the symmetry along the vertical centreline of the tube. It is also noted that the maximum axial velocity has been shifted toward the outer wall of the bend at an angular position as early as $\phi=22.5^\circ$ for the entire range of Dean numbers covered by Austin.

For $K=90$ in Fig. 3.2(a), a thin boundary layer is clearly seen at $\phi=45^\circ$. The development of the boundary layer seems to be dominant at $\phi \leq 90^\circ$. A secondary flow separation is shown near the inner wall of the bend for $\phi=45^\circ$. One sees such flow separation exists at the same location until $K=208$. The growth of the boundary layer can be justified by the increase of its thickness near the outer wall of the tube from $\phi=45^\circ$ to 90° at $K=90$ in Fig. 3.2(a). Inside the boundary layer, the downward movement of the fluid is clearly demonstrated by the boundary collision at the inner wall from $\phi=45^\circ$ to 90° in Fig. 3.2(a) through 3.2(c). Such boundary layer collision results in an upward stream in the core region moving toward the outer wall; and eventually develops into a familiar two-vortex flow pattern at the fully developed case.

For $K=208$ in Fig. 3.2(d), the upward stream has opened up forming a unique flow pattern at $\phi=90^\circ$. The flow pattern resembles two back-to-back J-shapes turned upside down. It is probably due to the interaction between the radial flow converging toward the centre of the tube, and the upward

stream moving the fluids at the inner bend toward the outer. The accelerated axial motion in the core results from the growth of the boundary layer all around the tube.

At $K=364$ in Fig. 3.2(e), one cannot see the occurrence of the boundary layer at $\phi=45^\circ$. The increase in curvature (ie, Dean number) has promoted the formation of secondary flow as shown on the flow patterns at $\phi=45^\circ$ in Fig. 3.2(a) through 3.2(d).

It is interesting to see the development of the secondary flow occurring near the inner bend. This phenomenon agrees quite well with the Austin's experimental result. He found that a second peak axial velocity profile occurring near the inner bend in addition to the first peak occurring near the outer bend. The first peak is due to the maximum velocity being crowded out from the centre toward the outer wall. Thus, the secondary flow is responsible for bringing fluids of larger velocity in the outer wall region to the inner. This results in the secondary flow, shown by the second peak in Austin's work, occurring toward the inner bend.

It is also noted that three types of secondary flow patterns for fully developed laminar flow in curved pipes are reported in the existing literature for small Dean numbers ($K \leq 20$), intermediate Dean numbers ($20 \leq K \leq 200$) and large Dean numbers ($K \geq 200$). In the intermediate Dean number range, it seems that the top boundary layer becomes thicker as it moves from $\phi=45^\circ$ to 90° as shown in Figs. 3.2(a)

through 3.2(d). Secondary flow does not develop immediately beyond the inlet for $K \leq 208$. At this stage, fluid motion seems to occur in the boundary layer only. Due to the influence of curvature, the fluid particles in the boundary layer will move from the outer wall to the inner until they collide. The line of symmetry along the vertical axis prevents any mixing between two halves of flow. The fluid in the boundary layer moves toward the outer wall at $\phi \geq 90^\circ$ or even at $\phi = 45^\circ$ if K is large enough. This is seen in Fig. 3.2(d) when $K = 208$. As shown in Figs. 3.2(a) through Fig. 3.2(c), the line of symmetry does not appear to cross the whole tube diameter at $\phi < 90^\circ$ in the developing flow. At $\phi = 90^\circ$, secondary flow patterns develop at the location above the tube centre. No vortex-like pattern can be found yet. Moreover, secondary flow patterns, which are formed due to the boundary layer collision near the inner wall, are seen to grow from the inner wall toward the outer wall which agrees well with the numerical predictions.

When the Dean number is large, such as in Figs. 3.2(e) through 3.2(h), no boundary layer is noticeable at $\phi \geq 45^\circ$. One can see a large increase in the magnitude of secondary flow vortices as the fluid moves further downstream from $\phi = 45^\circ$ to $\phi = 90^\circ$ in Fig. 3.2(e). In Figs. 3.2(e) through 3.2(h), vortex-form patterns are established near the inner wall at $\phi = 45^\circ$, which is in agreement with Austin's theory. As the fluid moves further downstream, secondary flow patterns appear to move toward the outer wall. At $\phi = 90^\circ$,

they appear to pick up more strength as is evident by the spiral-shape patterns seen below the tube centre. The spiral-shape patterns tend to grow in size as they move from $\phi=45^\circ$ to $\phi=90^\circ$. However, the size of vortex is small compared to the fully developed case. Finally, the secondary flow patterns will develop over the entire cross-section at $\phi=180^\circ$. The region near the inner bend is worth mentioning. An additional pair of vortices appears to develop near the inner wall at $\phi=90^\circ$ for $364 \leq K \leq 727$ in Figs. 3.2(e) through 3.2(h). In viewing the vorticity of the secondary flow in Figs. 3.2(e) through 3.2(f), the magnitude of secondary flow vortices increases as the Dean number increases. Though $Re=2300$ is shown in Fig. 3.2(h), it is believed that laminar flow still persists because of the curvature effect.

The effect of Dean number on the flow development at certain ϕ ($\phi=45^\circ$, 90° , 135° and 180°) is shown in Figs. 3.3, 3.4, 3.5 & 3.6 respectively. At $\phi=45^\circ$, a boundary layer can be observed for $K \leq 184$. At $K=184$ in Fig. 3.3, fluid particles in the boundary layer collide near the inner wall and as a result, an upward stream is formed moving toward the outer wall. All secondary flow patterns are still in the developing stage. The formation of secondary flow patterns, due to the boundary layer collision, begins at $K=184$ and develops near the inner wall only. At $\phi=45^\circ$, the development of secondary flow occurs at the inner wall region only. As shown in Fig. 3.3(h) for $Re=3747$, laminar

flow still persists at such a high Reynolds number.

At $\phi=90^\circ$ (Fig. 3.4), the boundary layer collision occurs early at $K=72$ whereas the formation of secondary flow patterns start at $K=99$ in the location above or at the tube centre. Austin shows that the second peak of the axial velocity will shift from the inner wall at $\phi=45^\circ$ toward the tube centre at $\phi=90^\circ$. Our development of secondary flow patterns agree quite well with his measurements. As shown in Figs. 3.4(f) and 3.4(g), the effects of Dean number seem to increase the magnitude of the secondary flow vortices and bring them closer together in the core. An additional pair of vortices, which begins to form at $K=624$ next to the inner wall, persists up to $K=827$.

At $\phi=135^\circ$ (Fig. 3.5), the secondary flow patterns seem to be well developed as they occupy almost the entire cross-section at $K=72$. A pair of symmetric vortices is seen at $K=572$. Again, the vortices appears to move closer to the tube centre as the Dean number increases from $K=572$ to 675 in Figs. 3.5(f) through 3.5(g) respectively. At $K=827$, the patterns become quite vague possibly because of the smoke diffusion at higher flow rate.

At $\phi=180^\circ$ (Fig. 3.6), fully developed flows are formed at $K\geq 500$. The line of symmetry is distorted at $K\geq 572$. The effect of Dean number (or Reynolds number) on the secondary flow patterns in the fully developed regime is not as significant as in the developing case. More details on the fully developed flow in a curved pipe can be found in

Chapter 2.

3.4.2 Secondary Flow Development at Bends with Uniform

Entrance Velocity Profile (Curvature ratio $d/2R_c=0.10$)

Fig. 3.7 shows the secondary flow development in the entrance region of a curved pipe at $K=94-827$. Once again, the top of each photograph represents the outside of the bend and the bottom represents the inside. A different smoke generating method was employed at this time. The length of entry flow for a uniform entrance velocity profile was given by Yao and Berger[8] as:

$$D = 5.73(e_1)(d)(2KR_c/d)^{1/2}$$

where, D =No. of degs. req'd for development

e_1 =Coefficient of entry length depending on the ratio of curvature, ($e_1=2.5$ for $d/2R_c=0.1$).

Based on the above correlation, it is noted that the range of parameters being studied in this experiment is still in a developing stage.

As shown by $\phi=45^\circ$ in Figs. 3.7(a) and 3.7(b), a small curvature effect, shown both by the magnitudes of K and ϕ , draws the fluid particles (light part) from the outer bend toward the inner in the boundary layer. One also sees the fluid particles (dark part) are pushed out from the inner bend toward the core. It is noted that the upward fluid

motion in the core cannot penetrate all the way to the outer wall. A flow separation is clearly seen occurring near the inner wall at $\phi=45^\circ$ for all Dean numbers. Secondary flow does not develop immediately beyond the inlet of the bend for $K \leq 99$. At $K=90$ in Figs. 3.7(a) and 3.7(b), a line separating the fluid particles into left- and right-hand regions is well defined, as the fluid particles are being pushed from the inner bend toward the outer bend. As the fluid moves further downstream, the development of secondary flow is a familiar two-vortex form with the fluid particles near the wall moving downward and the particles in the core moving upward. The secondary flow development at $K=260$ and 364 in Figs. 3.7(d) and 3.7(e) respectively are similar to the ones at $K=156$ in Fig. 3.7(c).

At $K=156$ in Fig. 3.7(c), secondary flow appears to exist in the inner bend region at $\phi=45^\circ$. Further downstream at $\phi=90^\circ$, the upward stream is opened up sideways. At $K=136$ to 827 (Figs. 3.7(c) to (h)), the collision of boundary layer at the inner wall region forces the formation of secondary flow patterns at $\phi=45^\circ$. Further downstream, such secondary flow patterns tend to move toward the outer wall and split sideways at the same time. More streamlines can be seen using the smoke generating method of burning paper straws in a smoke generating tube.

The change of flow development with Dean number effects at $\phi=0^\circ$, 45° , 90° , 135° & 180° are shown in Figs. 3.8, 3.9, 3.10, 3.11, & 3.12 respectively. At $\phi=0^\circ$ (Fig. 3.8), one

can see some disturbances due to the irregularities of the set up. A pair of very unstable vortices is shown near the inner wall at $Re=3271$ in Fig. 3.8(h).

Fig. 3.9 shows that the secondary flow patterns develop near the inner wall as K increases at $\phi=45^\circ$. A double peaked velocity profile similar to the one observed by Austin with parabolic entrance velocity profile was observed by Agrawal in his laser anemometer studies for flow development with uniform entrance velocity profile; as well as by Soh using numerical solution. Thus the secondary flow pattern is shifted toward the inner wall since a maximum velocity has developed near the outer wall due to the curvature effect. No vortex-like pattern can be seen at $\phi=45^\circ$ for $K \leq 127$ at $\phi=45^\circ$. Secondary flow development does not seem to play the major role in this mixing and mass transfer. It is noted that the size of secondary flow pattern decreases as Dean number increases.

Fig. 3.10 serves the same purpose as Fig. 3.9 for $\phi=90^\circ$. Here, one can note that the strength of the secondary flow pattern is much stronger than at $\phi=45^\circ$. Secondary flow patterns seem to grow in sizes from $\phi=45^\circ$ in Fig. 3.9 to $\phi=90^\circ$ in Fig. 3.10. As Dean number increases, the pair of vortex appears to shift toward the inner wall as well. Fig. 3.10(h) shows a pair of unstable vortices wandering around the outer wall at $Re=3911$ & $K=1244$. Laminar flow is believed to still occur at $Re=3911$ due to the presence of the secondary flow.

The opening of the secondary flow patterns is not as wide at $\phi=90^\circ$ as at $\phi=135^\circ$ (Fig. 3.11). Moreover, the secondary flow patterns tend to move toward the outer wall with a tremendous increase in strength as well. Only a mild split can be seen for the flow patterns shown at $\phi=180^\circ$ in Fig. 3.12. The line of symmetry can be maintained in all locations at all conditions. Dean number affects the shape of the secondary flow patterns. It is also believed that Dean number also affects the sizes of the secondary flow patterns.

3.5 Concluding Remarks

The developing secondary flow patterns induced by centrifugal forces in the entrance region of curved tubes with a curvature ratio of 0.10 are presented. Both parabolic and uniform entrance flows were included covering the Dean number range of $K=90-1244$ ($Re=283-3911$). Flow visualization was carried out at five equally spaced sections along the circumference at $\phi=0^\circ$ to 180° from the start of bend.

For small Dean number, boundary layer collision may be the drive for the development of secondary flow pattern in the entrance region of curved pipes. The spiral motion of the secondary flow patterns tend to stay below the tube centre for $\phi \leq 90^\circ$. Flow near the inner wall at $\phi=90^\circ$ is very complicated. This inner wall region for developing flow may be equivalent to the instability region which occurs near

the outer wall for the fully developed case.

These presented secondary flow patterns should be useful for comparison with predictions by numerical solutions.

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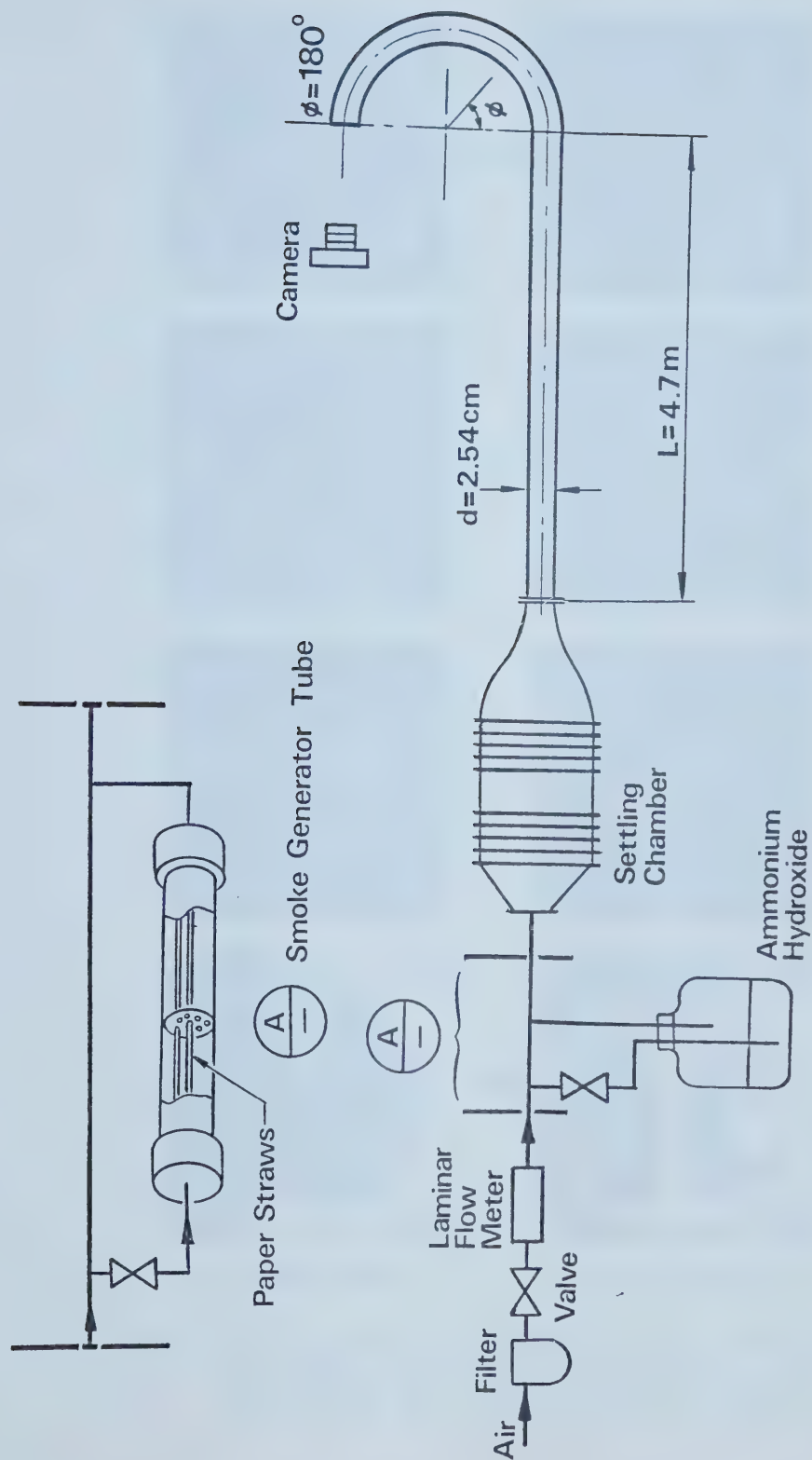


Fig. 3.1 Schematic diagram of experimental apparatus.

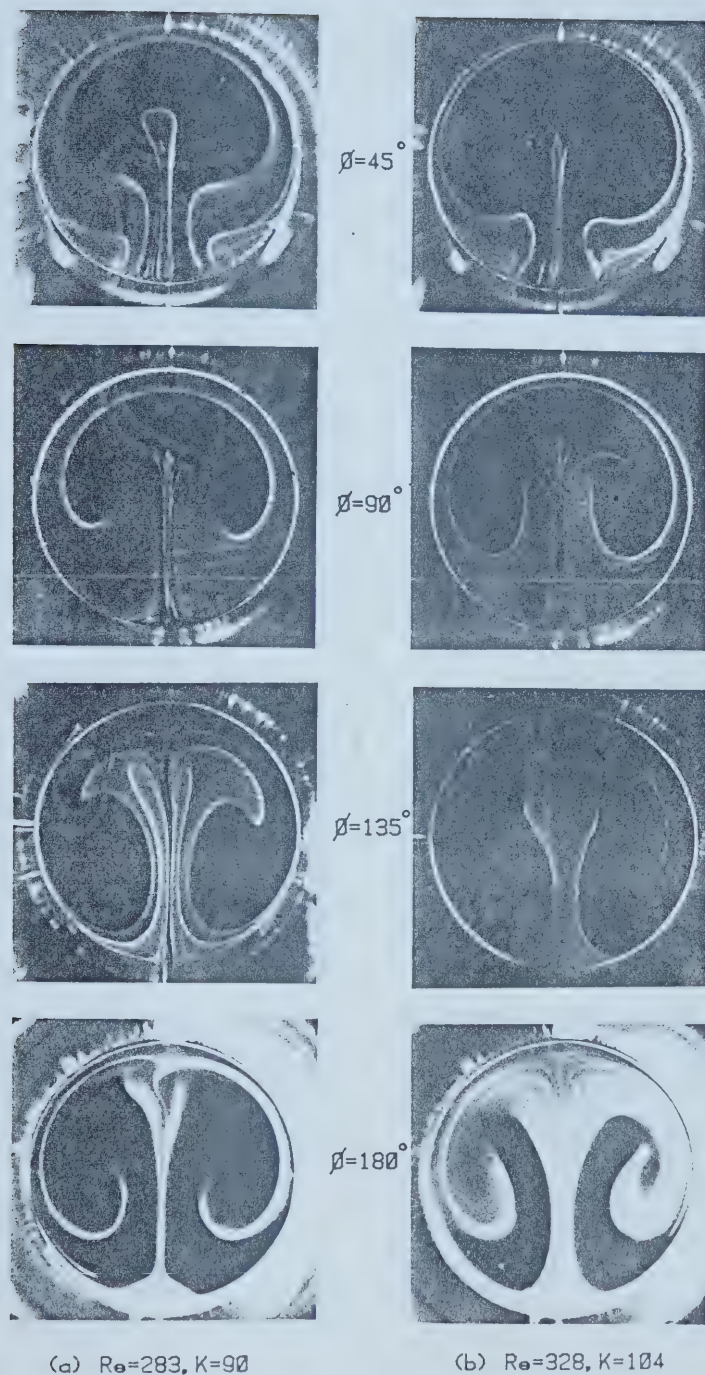


Fig. 3.2 Developing secondary flow patterns in the entrance region of a curved pipe with a fully developed parabolic entrance velocity profile.

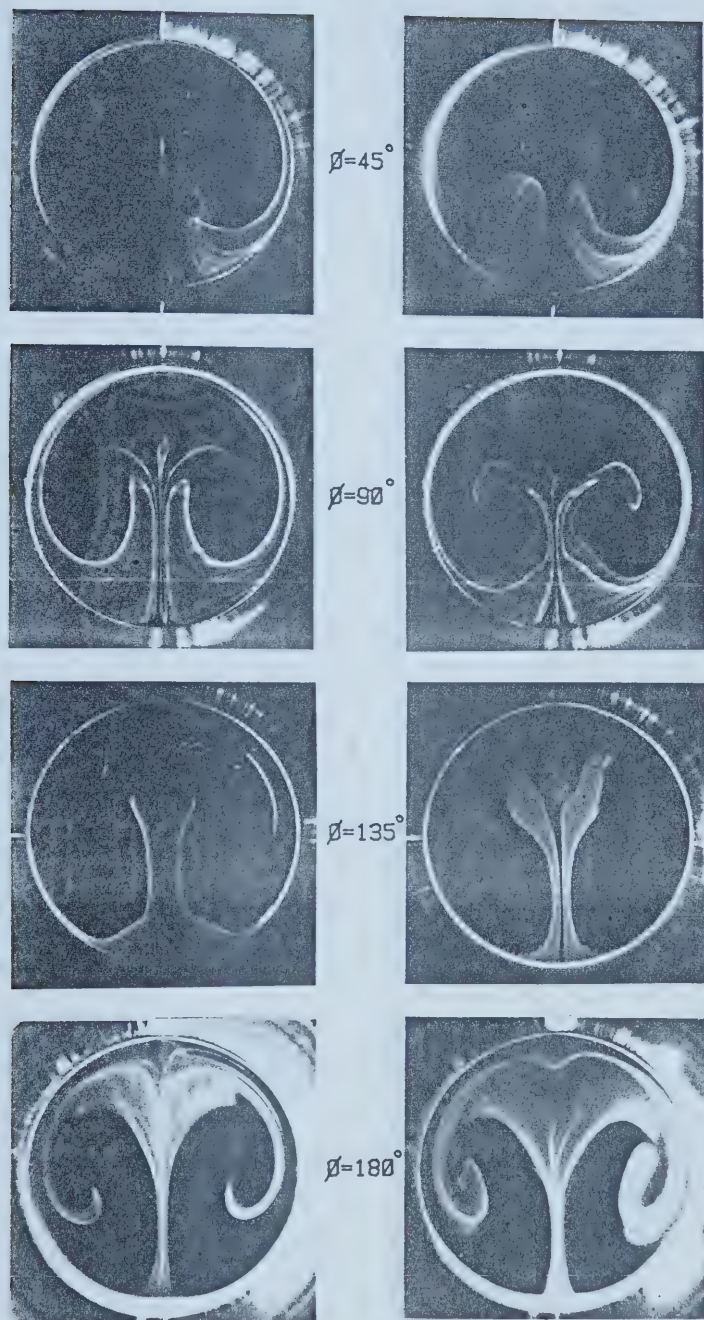
(c) $Re=493, K=156$ (d) $Re=657, K=208$

Fig. 3.2 Continued.

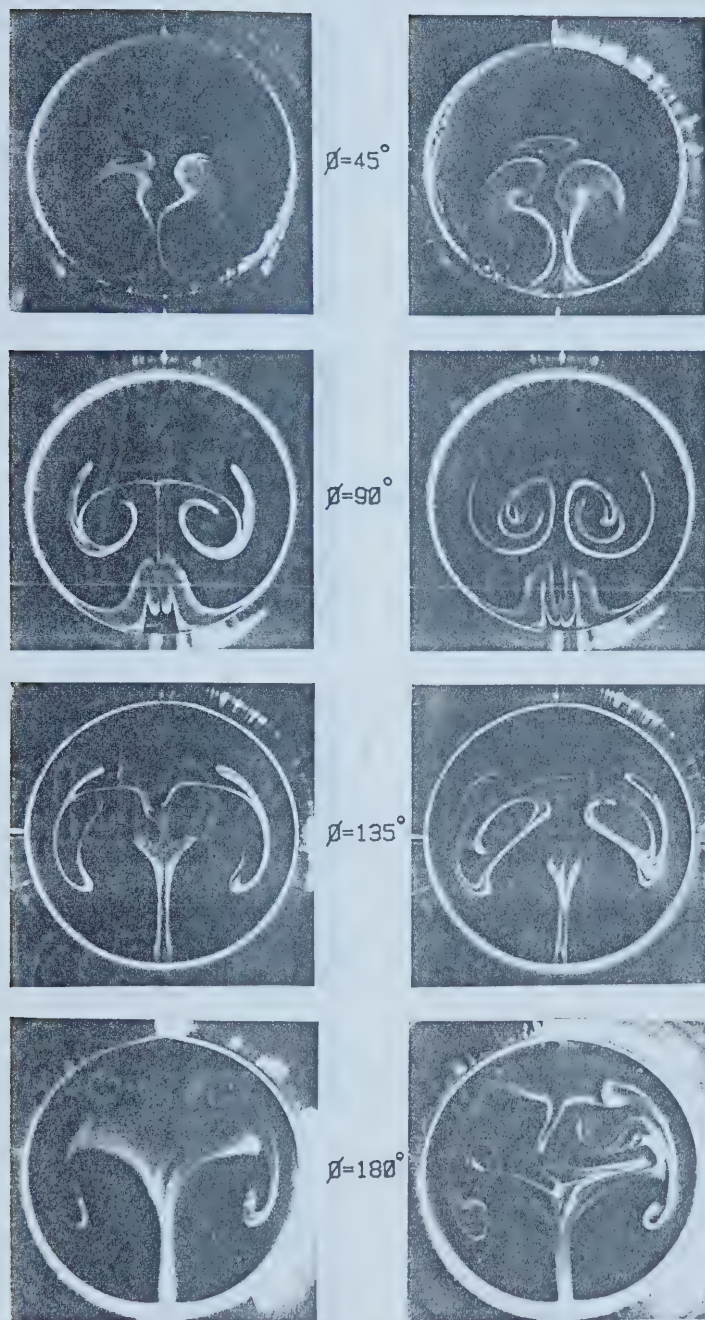
(e) $Re=1150, K=364$ (f) $Re=1643, K=520$

Fig. 3.2 Continued.

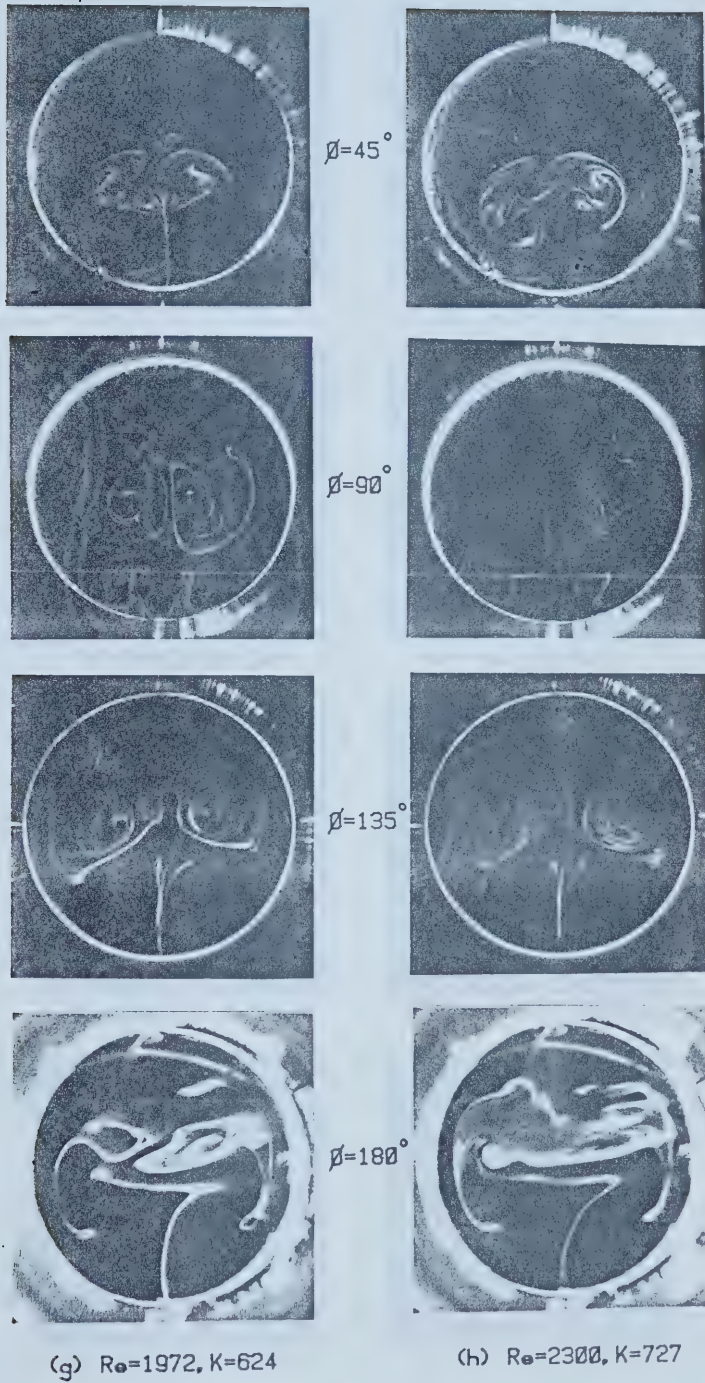


Fig. 3.2 Continued.

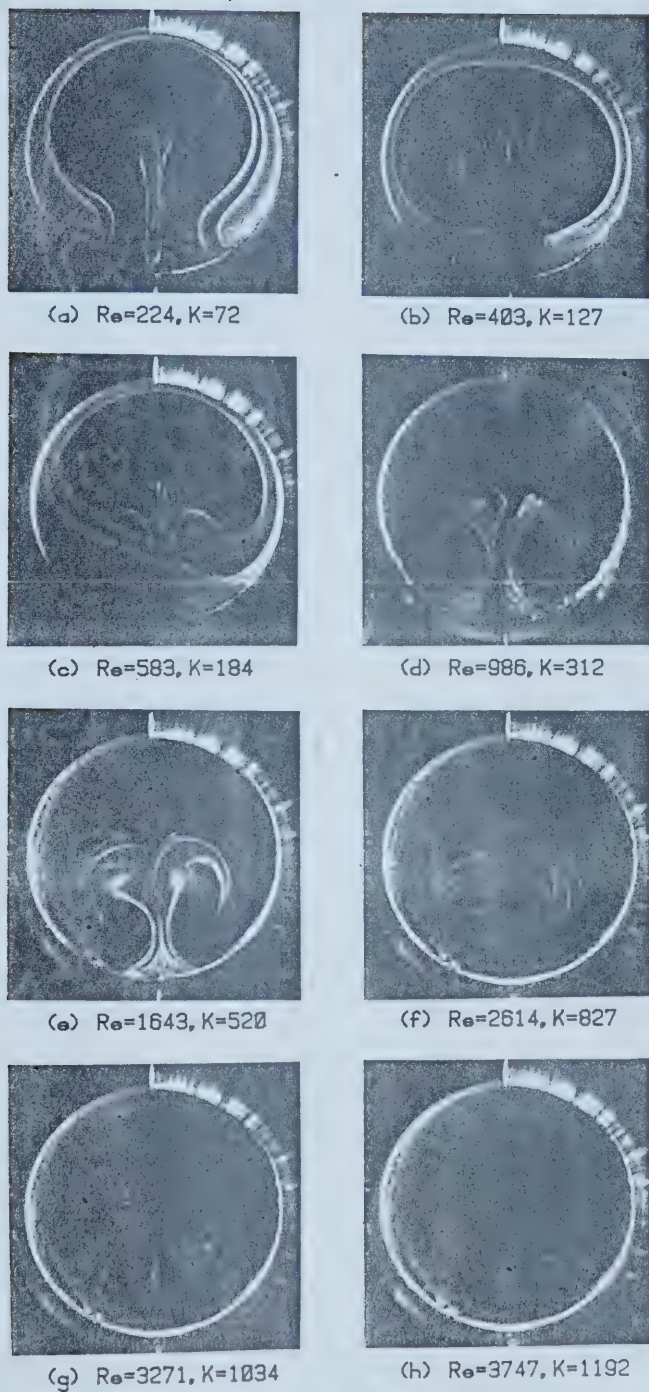


Fig. 3.3 The effect of Dean number on developing secondary flow patterns in a curved pipe (Parabolic at test section entrance), $\phi=45^\circ$.

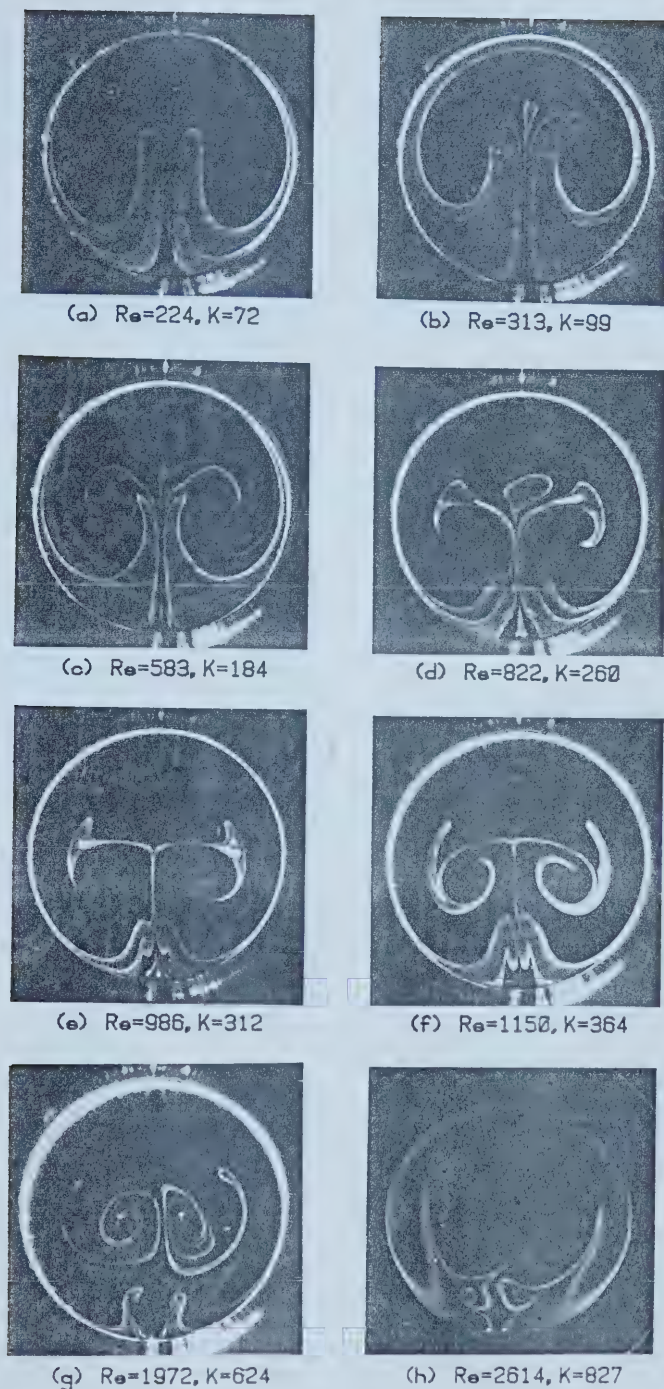


Fig. 3.4 The effect of Dean number on developing secondary flow patterns in a curved pipe (Parabolic at test section entrance), $\phi=90^\circ$.

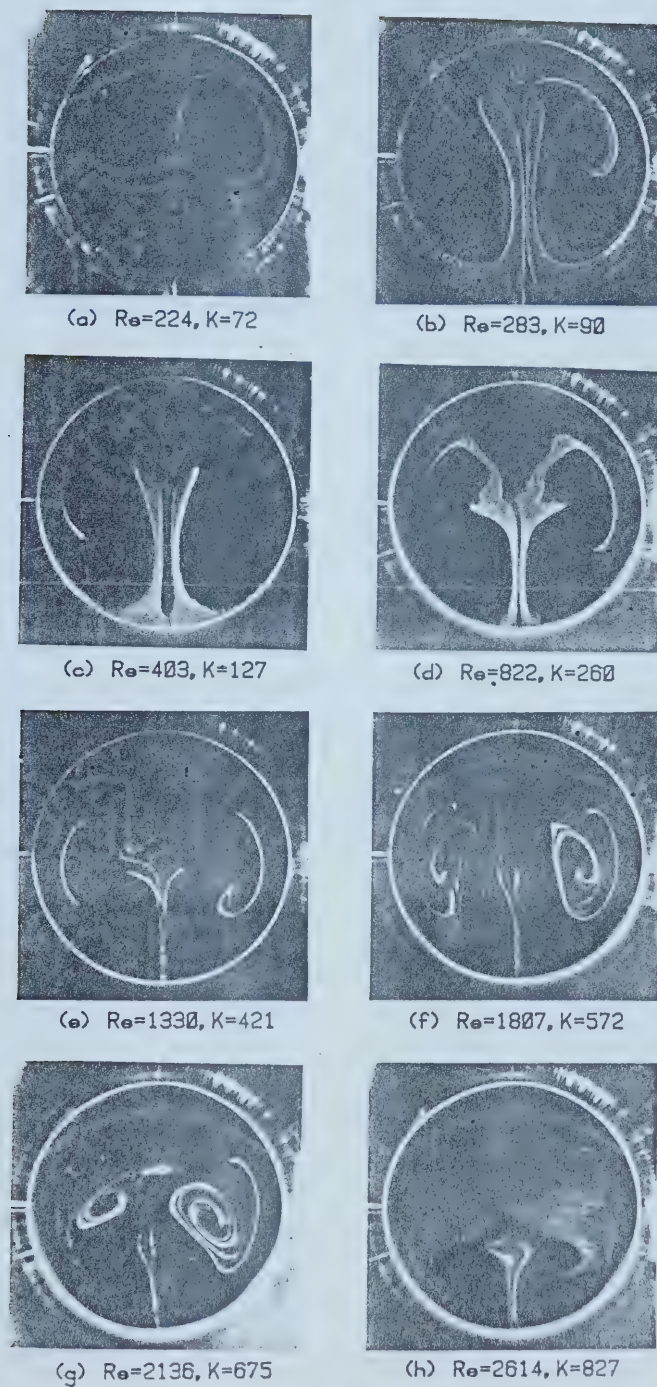


Fig. 3.5 The effect of Dean number on developing secondary flow patterns in a curved pipe (Parabolic at test section entrance), $\phi=135^\circ$.

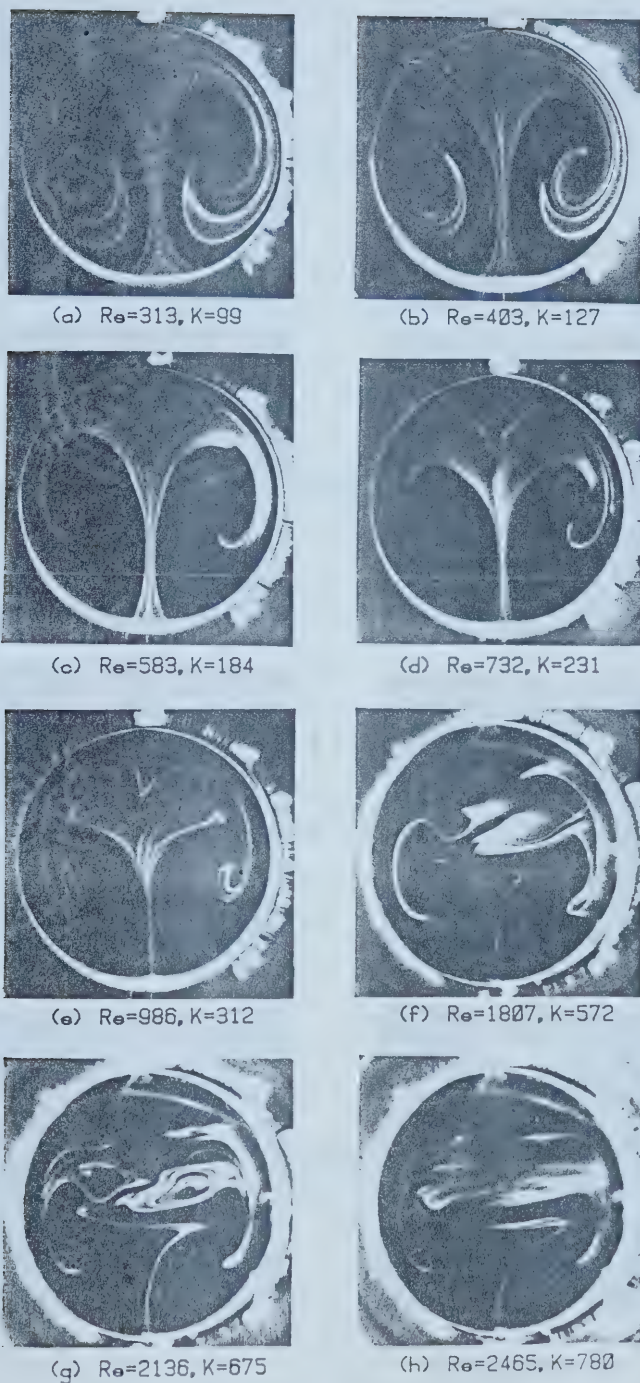


Fig. 3.6 The effect of Dean number on developing secondary flow patterns in a curved pipe (Parabolic at test section entrance), $\phi=180^\circ$.

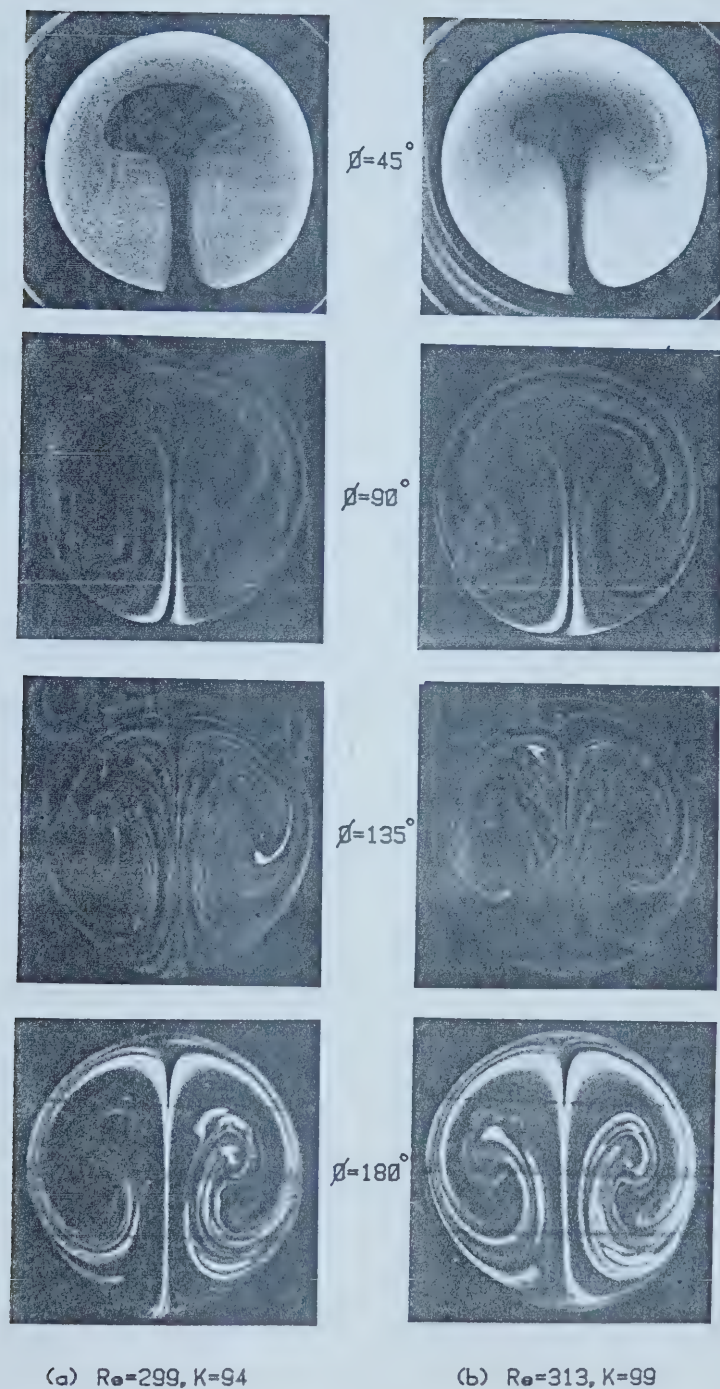


Fig. 3.7 Developing secondary flow patterns in the entrance region of a curved pipe with a uniform entrance velocity profile.

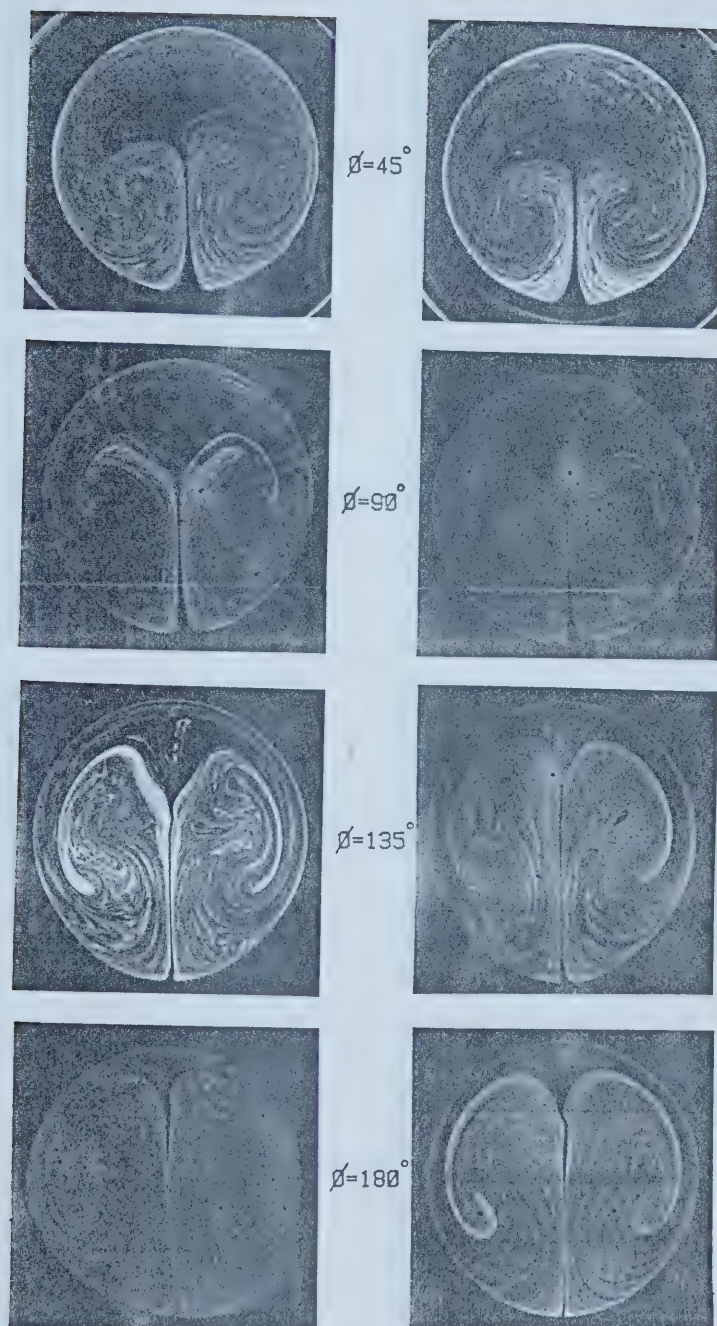
(c) $Re=493, K=156$ (d) $Re=822, K=260$

Fig. 3.7 Continued.

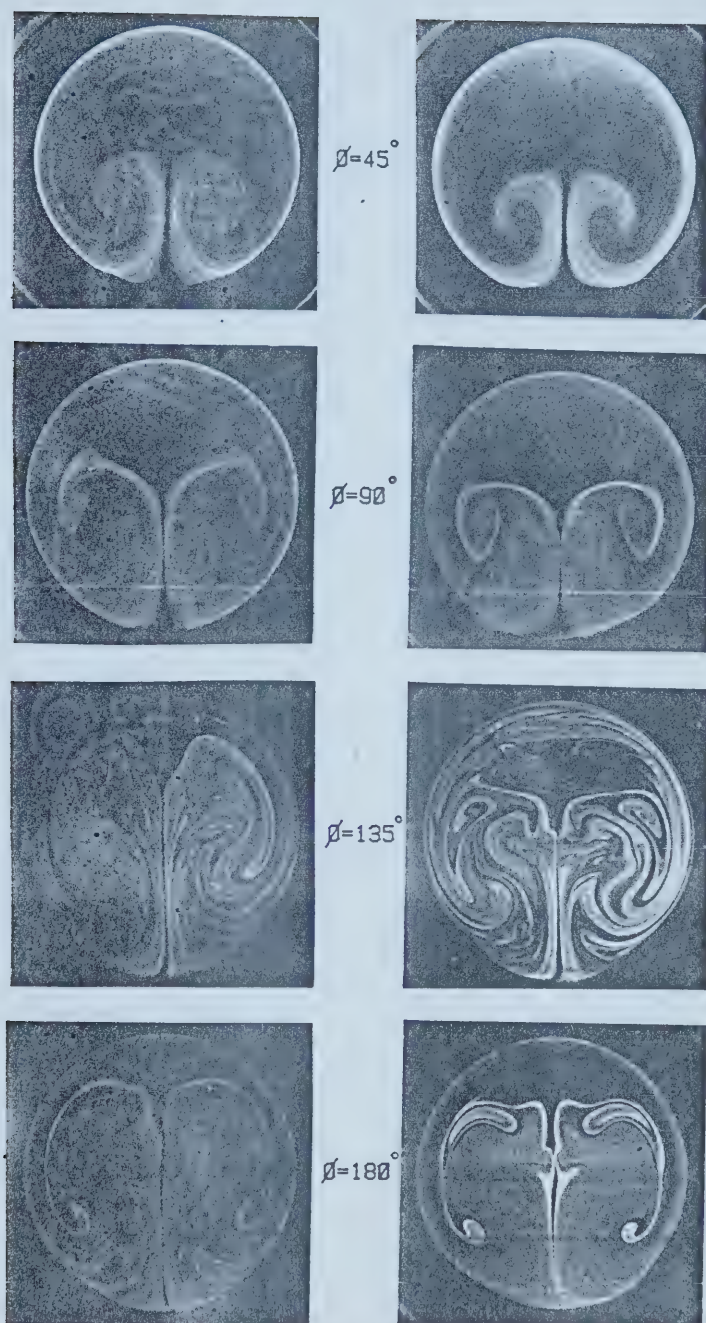
(e) $Re=1150, K=364$ (f) $Re=1643, K=520$

Fig. 3.7 Continued.

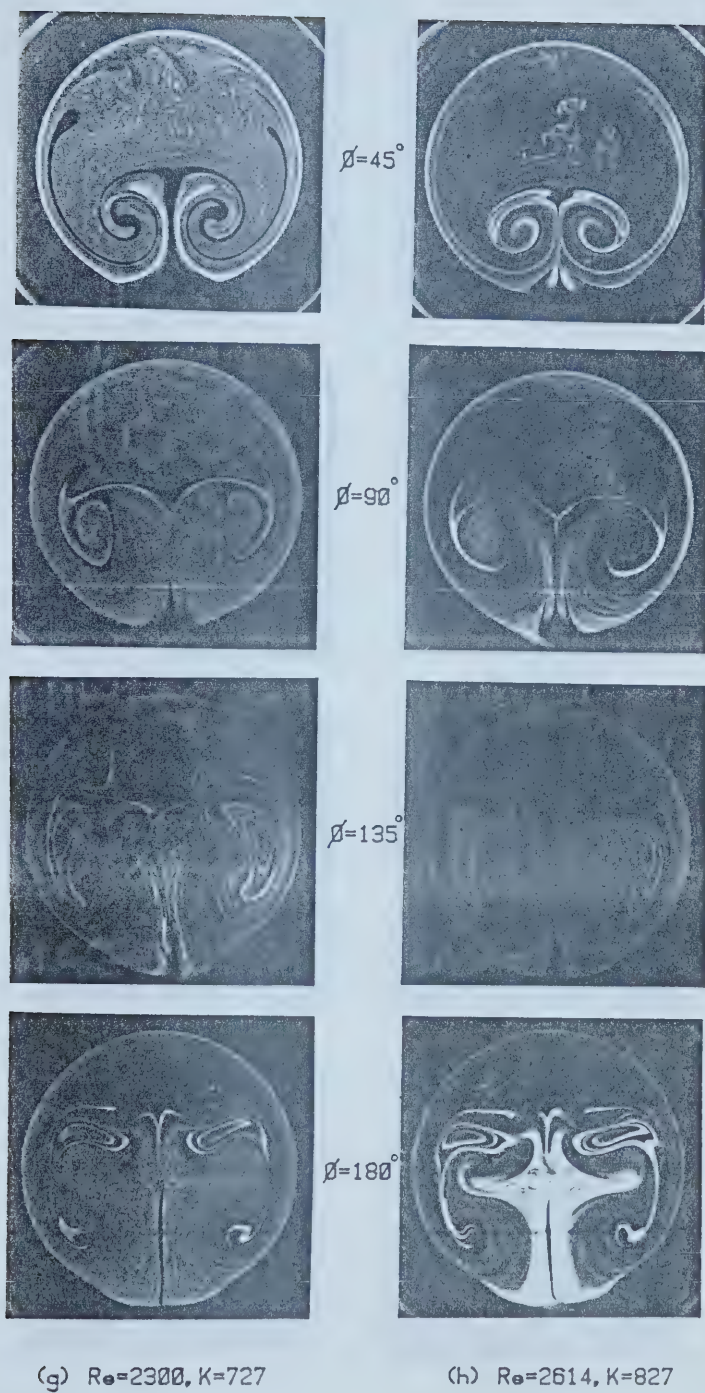


Fig. 3.7 Continued.

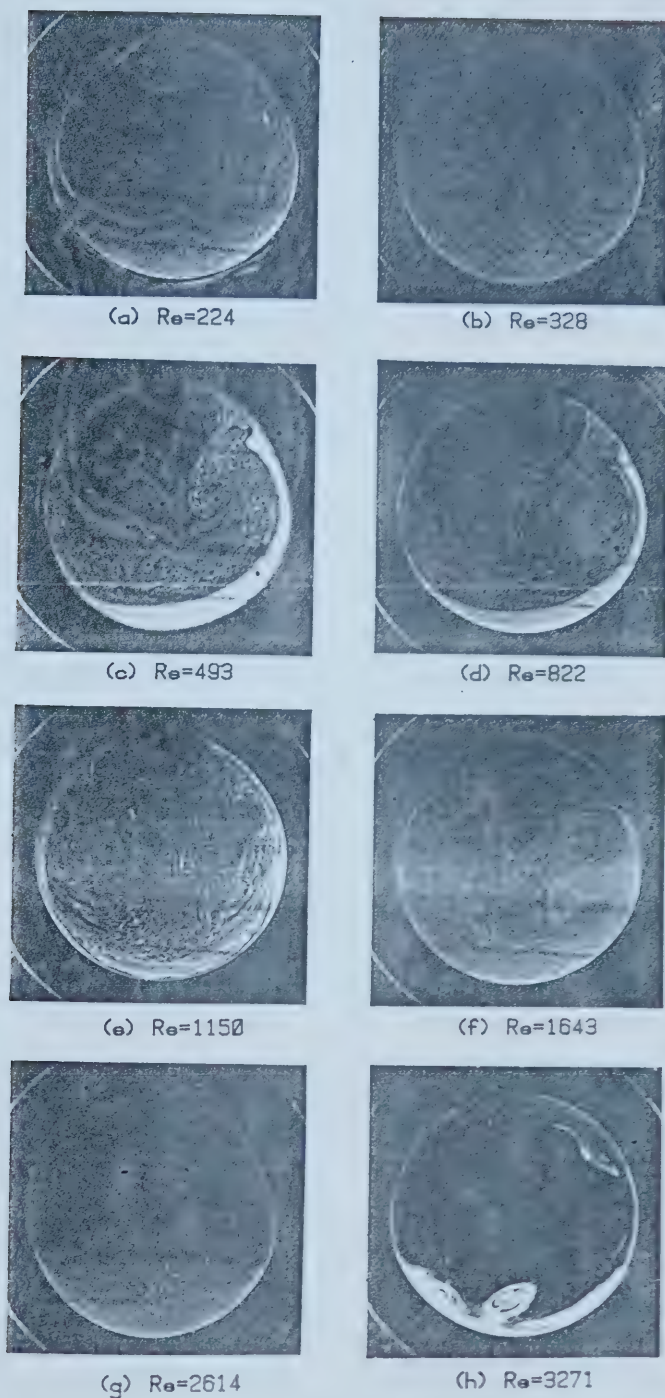


Fig. 3.8 Flow patterns at the exit of the settling chamber (Uniform at test section entrance), $\phi=0^\circ$.

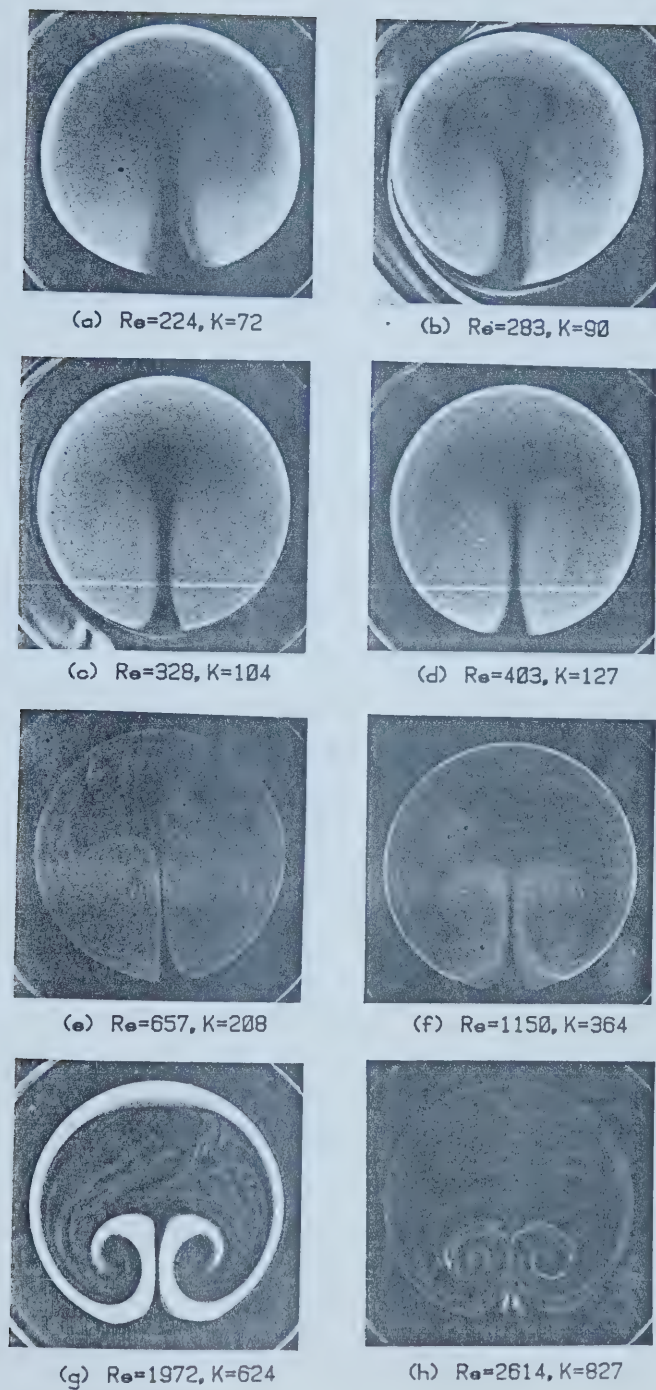


Fig. 3.9 The effect of Dean number on developing secondary flow patterns in a curved pipe (Uniform at test section entrance), $\phi=45^\circ$.

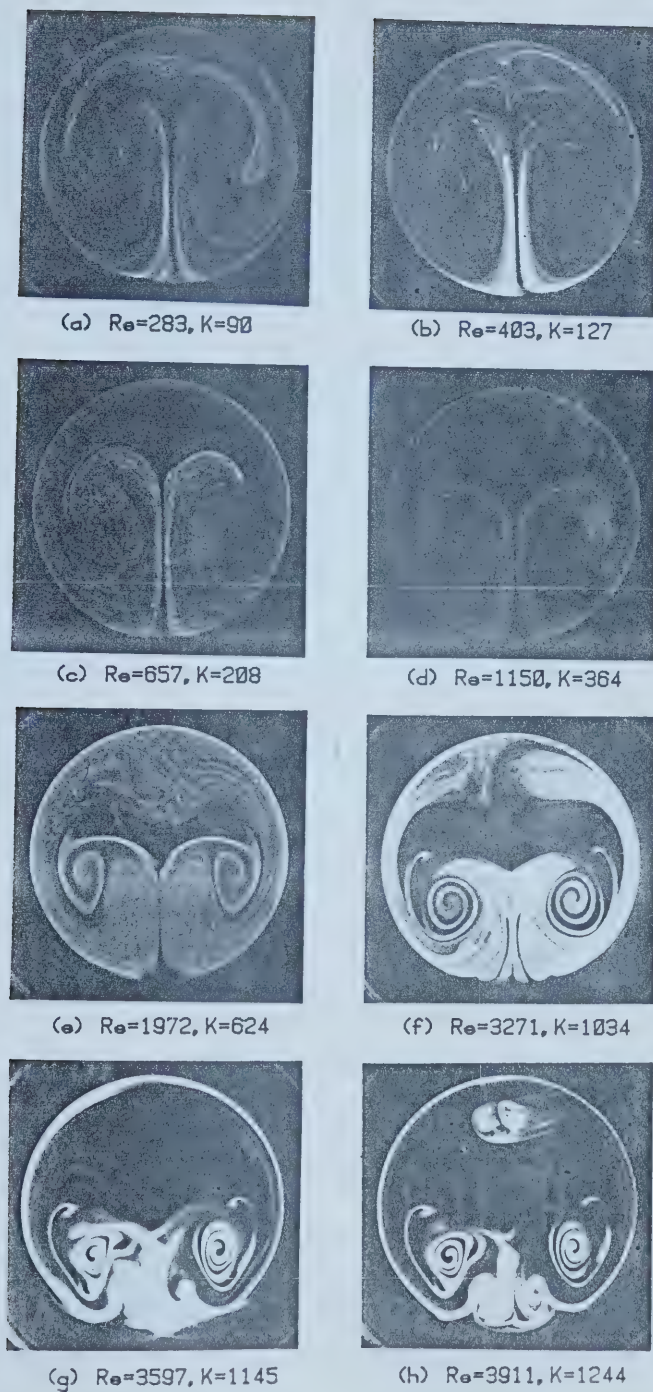


Fig. 3.10 The effect of Dean number on developing secondary flow patterns in a curved pipe (Uniform at test section entrance), $\phi=90^\circ$.



Fig. 3.11 The effect of Dean number on developing secondary flow patterns in a curved pipe (Uniform at test section entrance), $\phi=135^\circ$.

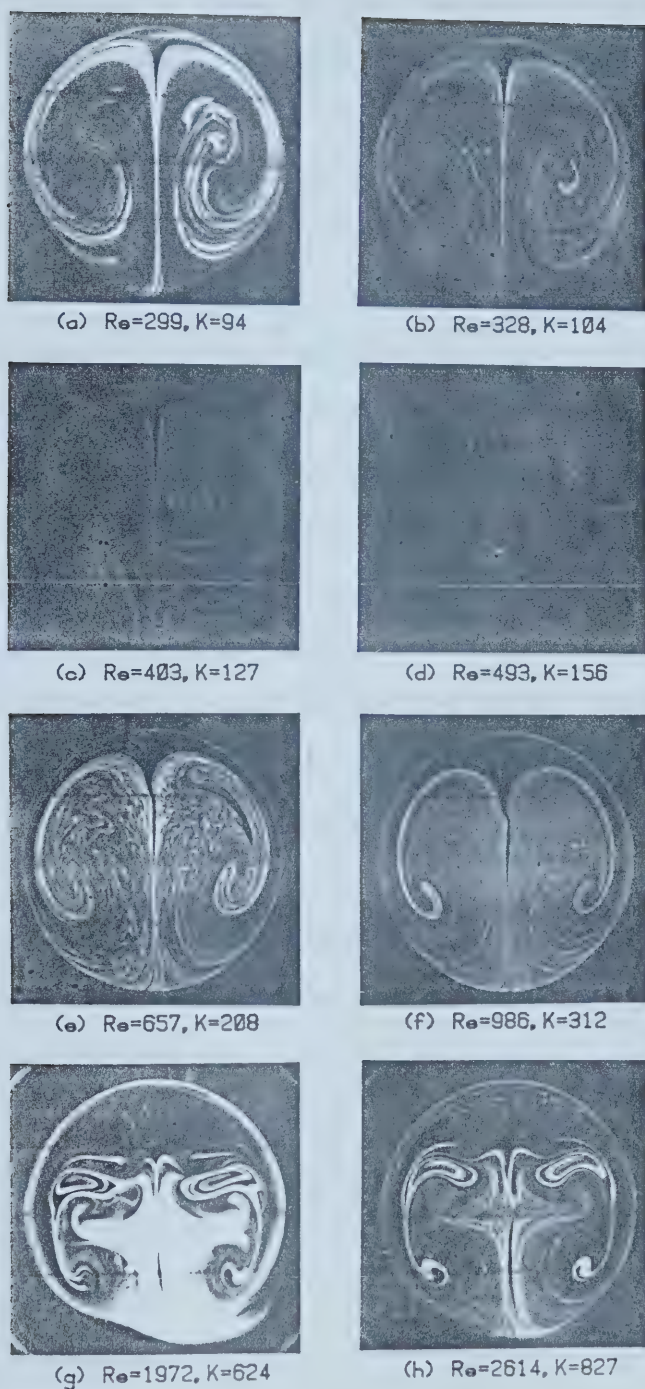


Fig. 3.12 The effect of Dean number on developing secondary flow patterns in a curved pipe (Uniform at test section entrance), $\phi=180^\circ$.

4. Steady Secondary Flow in the Thermal Entrance Region of Isothermally Heated Horizontal Tubes⁴

Summary

Photographs are presented for secondary flow patterns in isothermally heated horizontal tubes (tube inside diameters $d=2.63\text{cm}$ and $d=5.10\text{cm}$, and heated lengths $l=46.2\text{cm}$ and $l=52.5\text{cm}$ or 103.8cm , respectively) with free convection effect. Air at room temperature has been used as the flowing fluid. The developing secondary flow patterns in the thermal entrance region of isothermally heated horizontal tubes are shown for the dimensionless axial distance $z=0.8 \times 10^{-2}$ to 1.83 ($Re=3134-14$) and Rayleigh number Ra_1 from 3.65×10^4 to 5.18×10^5 for a range of constant wall temperatures $T_w=55-95^\circ\text{C}$. These secondary flow patterns are useful for future comparison with predictions from numerical solutions.

⁴A version of part of this chapter has been published. Cheng, K.C. and Yuen, F.P. ASME paper 84-HT-62 22nd ASME/AIChE National Heat Transfer Conf., Niagara Falls, New York, Aug. 5-8, 1984.

4.1 Introduction

The secondary flows caused by the buoyancy force in the thermal entrance region of isothermally heated or cooled tubes are known to be important in laminar forced convective heat transfer. The literature related to this area of study is quite extensive because of their importance in heat transfer equipment applications.

After the classical (pioneer) works of Graetz (1883) and Nusselt (1910) on pure forced convection in the thermal entrance region of isothermally heated tubes with fully developed laminar flow, the buoyancy effects on thermal entrance region problem, commonly known as Graetz problem, were studied by well-known investigators such as Drew[1], Colburn[2] and many others in the recent years. Their results were well summarized by McAdams[3]. For the Graetz problem with significant buoyancy effects in horizontal isothermal tubes, the classical method of solution is not applicable and one must employ a numerical solution[4]. For the uniform wall temperature boundary condition the buoyancy effect begins in the Leveque solution region near the thermal entrance. Rayleigh number is the governing parameter[4]. When the bulk fluid temperature approaches the wall temperature in a fully developed forced convection region, the asymptotic Nusselt number equals $Nu_{\infty}=3.66$. Because of this heat transfer mechanism, the correlation of heat transfer data on laminar mixed convection in an isothermal horizontal tube[5] is very difficult. It is

believed that flow visualization studies on developing secondary flow patterns in the thermal entrance region of an isothermally heated horizontal tube with free convection effects will provide some insight into the heat transfer mechanism. The results will complement the existing theoretical and experimental investigations reported in the literature.

The objective of this chapter is to investigate developing secondary flow patterns in the thermal entrance region of isothermally heated horizontal tubes with significant free convection effects. The flow visualization was facilitated by injecting smoke into air. The secondary flow patterns for combined free and forced convection in isothermal horizontal tubes shall provide an insight into the heat transfer mechanism.

4.2 Experimental Parameters

The experimental parameters for isothermally heated horizontal tubes are Reynolds number, Rayleigh numbers $Ra_1 = g\beta(T_w - T_o)d^3/\nu\kappa$, $Ra_2 = g\beta(T_w - T_c)d^3/\nu\kappa$, and dimensionless axial distance $z = (l/d)/RePr$. For the first series of experiments, $d = 2.63\text{cm}$, $l = 46.2\text{cm}$, $T_w = 55\text{--}95^\circ\text{C}$, $z = 0.8 \times 10^{-2} \text{--} 1.83$, $Re = 14\text{--}3134$, $Ra_1 = 3.65 \times 10^4 \text{--} 6.30 \times 10^4$ and $Ra_2 = 8.64 \times 10^3 \text{--} 4.67 \times 10^4$. For the second series of experiments, $d = 5.10\text{cm}$, $l = 52.5\text{cm}$ or 103.8cm , $T_w = 56\text{--}94^\circ\text{C}$, $z = 1.7 \times 10^{-2} \text{--} 3.7 \times 10^{-1}$, $Re = 77\text{--}1686$, $Ra_1 = 2.94 \times 10^5 \text{--} 5.18 \times 10^5$, $Ra_2 = 9.07 \times 10^4 \text{--} 3.35 \times 10^5$ and the ratio of Grashof number to

the square of Reynolds number $\Gamma=0.206-98.60$.

4.3 Experimental Apparatus and Procedure

The experimental apparatus for isothermally heated straight horizontal tubes is shown schematically in Fig. 4.1. For combined free and forced convection tests in a horizontal tube, a copper tube (inside diameter $d=2.54\text{cm}$, heating length $l=46.2\text{cm}$) with heating jacket (double-pipe heat exchanger) was used and hot water was circulated at high speed from a constant temperature bath to the test section in the counterflow direction. The heating loop, which supplies hot water to the heating jacket, consists of an insulated temperature controlled hot water tank (389 litres capacity) with a pump, rotameter and a by-pass for flow rate control. Fig. 4.2 shows a typical calibration curve for the rotameter installed at the outlet of the constant temperature bath. To promote turbulence in the heat exchanger, a coil was inserted in the annular space. The test section was insulated to prevent heat loss. The inlet and outlet bulk temperatures of the heating fluid (hot water) to the heating section were measured using 0.7 mm o.d. sheathed iron-constantan thermocouples. The inlet and outlet centreline air temperatures in the test section were measured by type T thermocouples (dia.=0.254 mm). All thermocouples were initially calibrated using a quartz thermometer. The maximum error was $\pm 0.3^\circ\text{C}$. Fig. 4.3 shows a typical calibration curve for the thermocouples. It was

confirmed that the inlet and outlet hot water temperature temperatures are nearly equal and the average temperature was taken as the average tube wall temperature T_w . Smoke was generated by burning straw paper sticks inside a copper smoke generator tube (see Fig. 4.1). The smoke generator tube was located before the settling chamber. Secondary flow patterns were observed at the exit of the isothermally heated section where the heated air was discharged into atmosphere. A slit light source was provided by a slide projector. Room compressed air was used and the air flow rate was measured by a Meriam laminar flow element. The entrance length after the settling chamber was 3.2m for the first series of experiments, and 6.2m for the second series to ensure fully developed laminar flow at the inlet of the heated section. A glass plate which was placed between the camera and the exit of the test section was used to protect the camera lens.

4.4 Results and Discussion

For the first series of experiments, the heated tube length was fixed at $l=46.2$ cm ($l/d=18.1$). A large number of photographs were obtained covering the constant wall temperature range $T_w=55-93^\circ\text{C}$ and Reynolds number range $Re=14-3134$. However, only typical examples of secondary flow patterns are shown in Figs. 4.4 to 4.7 for constant wall temperatures $T_w=55-56$, 65 , $74-76$ and $92-94^\circ\text{C}$, respectively. One sees the effects of buoyancy forces on

developing secondary flow patterns in the thermal entrance region of isothermally heated horizontal tubes from the photographs in which the dimensionless axial distance z , Reynolds number, and Rayleigh numbers Ra_1 , Ra_2 are noted. It is assumed that thermal properties of the fluid do not vary with temperature. Also, no provision was made to prevent axial heat conduction through the copper tube wall from the heated test section in the upstream direction. Further references on combined free and forced laminar convection in the thermal entrance region of an isothermally heated horizontal tube can be found from most recent works [6-9].

4.4.1 Photographs for $T_w=55-65^\circ\text{C}$ and $Ra_1=3.65-4.37 \times 10^4$, Figs. 4.4 and 4.5

The photographs are arranged in the order of increasing dimensionless axial distance z from the thermal entrance ($z=0$). Thus, the Reynolds and Rayleigh numbers (Re and Ra_2) vary with z . In Fig. 4.4, the changing secondary flow patterns with z are of particular interest. The secondary flow patterns in Figs. 4.4(a) to (d) are qualitatively similar and are characterized by a thin ascending boundary layer along the tube wall and a fairly complicated flow in the core region. Because of a rather large value of $ReRa_2=(5.69-9.31) \times 10^7$, the flow is still believed to be laminar, but flow in the core region is unstable for $z \leq 0.014$. Figs. 4.4(e) to (h) reveal the different features

for secondary flow patterns. For the range $z=1.4 \times 10^{-2}$ to 3.3×10^{-2} , the secondary flow in upper part is clearly divided into left- and right-hand regions whereas the demarcation line for the lower part is not so clear probably due to the mixing effect near the lower wall. The secondary flow was found to be stable for $z \geq 1.4 \times 10^{-2}$. The secondary flow patterns shown in Figs. 4.4(i) and (j) are familiar ones except that the secondary flow does not seem to exist near the lower wall due to rather uniform air temperature there. This trend is further magnified in the downstream region $z \geq 1.63 \times 10^{-1}$. Although it is not shown because of the rather weak secondary flow, a pair of weak vortices is still observed near the upper wall at $z=1.83$, $Re=14$ and $Ra_2=8.94 \times 10^3$. Further downstream $z > 1.83$, the temperature field may be regarded to be fully developed with a complete disappearance of the secondary flow. When the secondary flow is confined to the region near the upper wall in the form of a pair of vortices, the heated secondary flow leaves as a plume upwards and this particular feature is illustrated in Fig. 4.5 for $z=1.63 \times 10^{-1}$ and 4.48×10^{-1} . In interpreting the secondary flow patterns, one must recognize the roles of Reynolds and Rayleigh numbers since the heated tube length is fixed.

4.4.2 Photographs for $T_w=74-76^\circ\text{C}$ and $Ra_1=5.22 \times 10^4$, Fig. 4.6

Fig. 4.6(a) reveals a thin ascending boundary layer near the side wall and a hot air current seems to rise from

the lower wall covering a large core region. The situation is somewhat similar for Fig. 4.4(b). The trends of secondary flow patterns for Figs. 4.6(c) to (h) are similar to those shown in Fig. 4.4 for $z=1.4 \times 10^{-2}$ to 3.3×10^{-2} but with larger free convection effect. At $z=1.83$, $Re=14$, $Ra_2=1.48 \times 10^4$ one still observes a very weak secondary flow in a small region near the upper wall.

4.4.3 Photographs for $T_w=92-94^\circ\text{C}$ and $Ra_1=6.30 \times 10^4$, Fig. 4.7

The general features of the secondary flow pattern for different values of z are similar to those shown in Fig. 4.6 with larger free convection effect for $Ra_1=6.3 \times 10^4$ and $Ra_2=1.23 \times 10^4-4.67 \times 10^4$.

In Figs. 4.4 to 4.7, one may gain more insight regarding the free convection effect by comparing the secondary flow patterns for the identical dimensionless axial distance z (or Reynolds number) for different wall temperatures. For example, the secondary flow patterns appear to be different in Figs. 4.4(i) and 4.7(i) at $z=0.054$ or in Figs. 4.4(j) and 4.7(j) at $z=0.082$. At higher Rayleigh number, the fully developed temperature field is approached earlier at smaller value z . This observation agrees with the theoretical results for large Prandtl number fluid [4]. Since the heated test tube length is fixed, the secondary flow patterns near the thermal entrance ($z=0$) after the onset of free convection effect cannot be obtained in this flow visualization study.

4.4.4 Photographs for $T_w=56^\circ\text{C}$ and $Ra_1=2.94-3.08 \times 10^5$, Figs. 4.8 & 4.9

For the second series of experiments, the heated tube lengths were either $l=52.5\text{cm}$ ($l/d=10.3$) or $l=103.8\text{cm}$ ($l/d=20.4$). Flow visualization studies on developing secondary flow patterns in the thermal entrance region of an isothermally heated tube are made possible for the range of $Ra_1=2.94 \times 10^5-5.18 \times 10^5$ and $Ra_2=9.07 \times 10^4-3.35 \times 10^5$. Comparing this Ra range to the first series of tests (Figs. 4.4 to 4.7), it is noted that much higher Ra range was obtained which implies that more buoyancy forces were involved. Thus, the free convection effect is relatively higher in this experiment at the same flow rate. Each photograph includes the dimensionless axial distance z , Reynolds number, Rayleigh numbers Ra_1 & Ra_2 as well as the ratio of Grashof number to the square of Reynolds number, Γ . The parameter Γ represents only the ratio of free convection effect over the forced laminar convection near the thermal entrance ($z=0$). In interpreting the results, it is noted that $z=0.05$ the local Nusselt number already approaches the limiting value $Nu_\infty=3.66$ based on Graetz solution without free convection effects. The upstream region $z=0-10^{-4}$ may be regarded to be the Leveque solution region and is outside the scope of present study. Since the heating length did not vary during each test, one should recognize the roles of Reynolds and Rayleigh numbers in interpreting the secondary flow patterns. Polyvinyl chloride (PVC) tube was used at

the leading tube section to minimize axial heat conduction from the heated section to the upstream direction.

Figure 4.8 includes photographs taken with the application of the first shell of heating jacket ($l=52.5\text{cm}$), whereas Figure 4.9 shows photographs with both shells being used ($l=103.8\text{m}$). In interpreting Figs. 4.8 and 4.9, one must recognize the rules of Re and Ra .

At a constant flow rate, Fig. 4.9, when compared with Fig. 4.8, shows a developing secondary flow pattern with larger free convection effect. See Fig. 4.8(b) and Fig. 4.9(b) as an example. At a constant z , Fig. 4.9, when compared with Fig. 4.8, shows the flow patterns with lesser free convection effect (Figs. 4.8(d) and Fig. 4.9(b)). Due to the higher Ra , flow patterns are completely different from the results in Figs. 4.4 to 4.7.

4.4.4.5 Photographs for $T_w=74-75^\circ\text{C}$ and $Ra_1=4.16-4.27 \times 10^5$,

Figs. 4.10 & 4.11

Photographs in each figure are arranged in the order of increasing axial distance z from the thermal entrance. Unlike photographs presented in the first series, Fig. 4.10(a) shows no distinction between the boundary layer and core region at $z=1.1 \times 10^{-2}$. At $z=0.011$ in Fig. 4.10(a), the line separating left- and right-hand regions barely passes the tube centre. As z increases further, this separating line extends downward to the bottom wall until the flow pattern becomes two counter-rotating vortices as shown in

Figure 4.10(f). When $z > 0.027$, the two-vortex flow are lifted up, leaving a crescent-shaped region at the lower section of the tube. At $z = 0.095$ (Fig. 4.10(j)), a small pair of vortices still exists near the top wall. For $z > 0.095$, a fully developed forced convection is believed to have achieved with Nusselt number approaching $Nu_{\infty} = 3.66$ while the secondary flow pattern will completely disappear. It is noted that the secondary flow patterns obtained in $0.011 \leq z < 0.027$ of Fig. 4.10 do not exist in our earlier study. Fig. 4.11 shows similar flow patterns vortex flow will rise, leaving a crescent-shaped secondary flow pattern due to the buoyancy effect. It is understood that the counter-rotating vortices will eventually be formed and separated by the line of symmetry. However, Figs. 4.10(a) to (e) and Figs. 4.11(a) to (d) show that the line of symmetry cannot penetrate all the way down to the bottom wall. The line separating different regions will flow downward due to the upward stream near the wall and be deflected sideways and later upward when it encroaches the ascending streams near the wall. This is particularly the case when z is small. The secondary flow pattern will initially have a sharp turn occurring along the wall, but will enlarge to become a smooth curve as z increases from $z = 0.017$ to 0.037 in Fig. 4.11. See Figs. 4.10(c) and (d), and Figs. 4.11(c) and (d) for the illustration. The trend of secondary flow patterns at $z > 0.027$ is similar to those shown in Figs. 4.8 and 4.9 at $T_w = 55^{\circ}\text{C}$ except with larger

free convection effect.

4.4.6 Photographs for $T_w=94^\circ\text{C}$ and $Ra_1=5.17-5.18 \times 10^5$, Figs. 4.12 & 4.13

Figs. 4.12 and 4.13 show secondary flow patterns with $Ra_1=5.17-5.18 \times 10^5$ and $Ra_2=1.30 \times 10^5-3.35 \times 10^5$. The general features of secondary flow patterns for different values of z are similar to those shown in Figs. 4.10 and 4.11, except the free convection effect is more pronounced. One may gain more insight regarding the free convection effect by comparing the secondary flow patterns for the identical axial distance z and different Ra_1 & Ra_2 in Figs. 4.8 through 4.13. At higher Rayleigh number, the fully developed temperature field is approached earlier at a smaller value of z . This observation has been confirmed in our earlier study and agrees with the theoretical results for large Prandtl number [4]. Once again the onset of the free convection effect cannot be obtained in these experiments since the heated tube lengths are fixed.

4.5 Concluding Remarks

The developing secondary flow patterns in the thermal entrance region of an isothermally heated horizontal tube are presented for $z=0.8 \times 10^{-2}$ to 1.83 ($Re=3134-14$) for a range of constant wall temperatures $T_w=55-95^\circ\text{C}$ with unheated air temperature at about 25°C . The photographs provide some insight into heat transfer mechanism due to buoyancy effect.

It is believed that some of the secondary flow patterns shown have not been reported in the literature and should be compared with predications from future numerical solutions.

4.6 References

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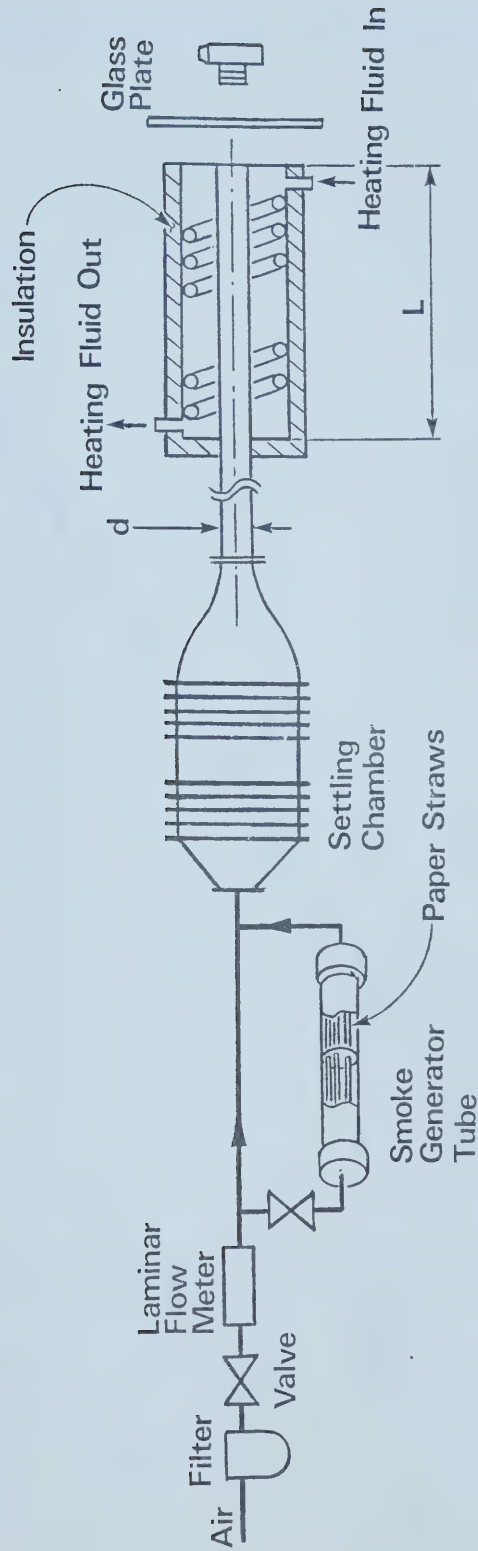


Fig. 4.1 Schematic diagram of experimental apparatus.

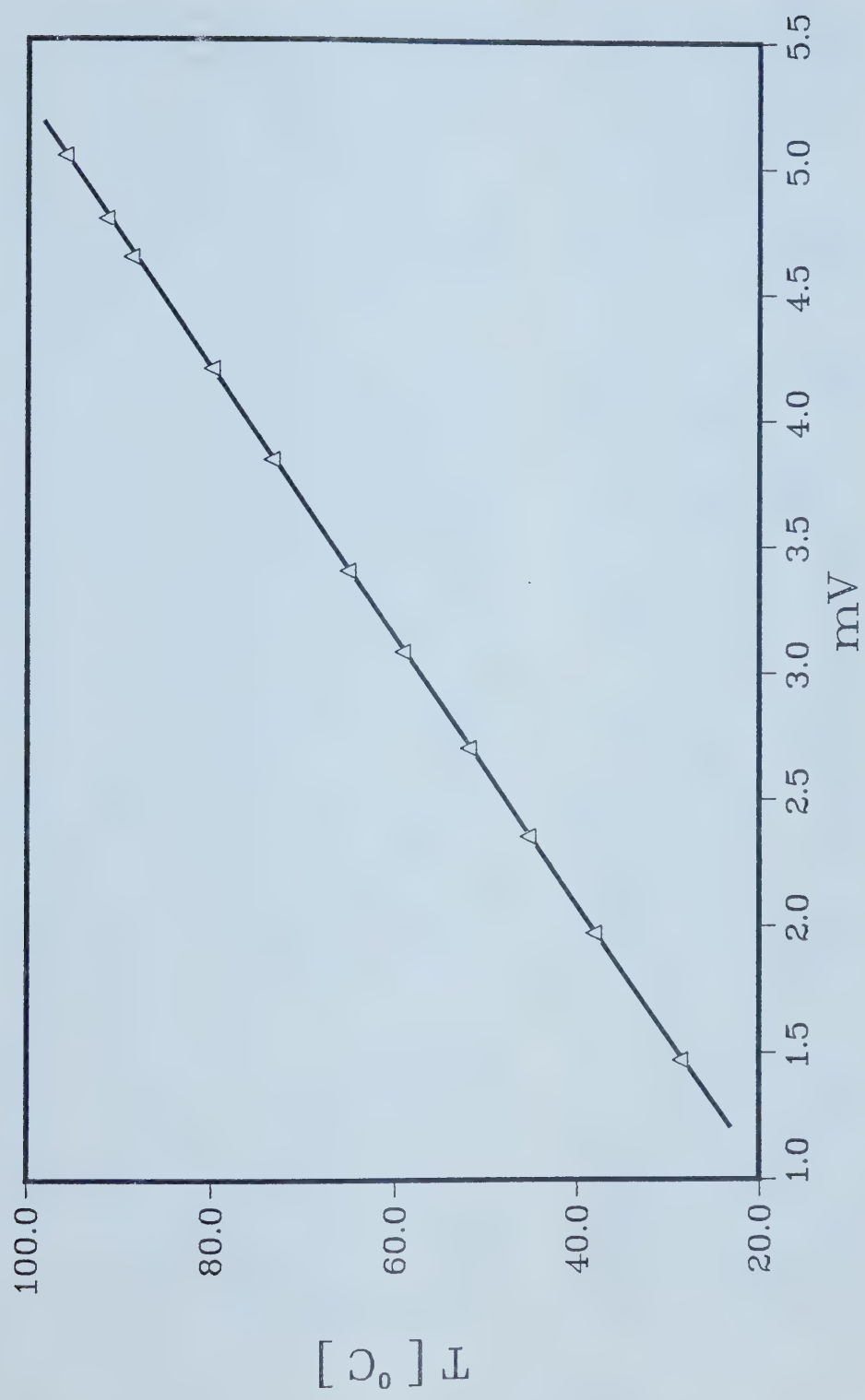


Fig. 4.2 A typical calibration curve for the thermocouples.

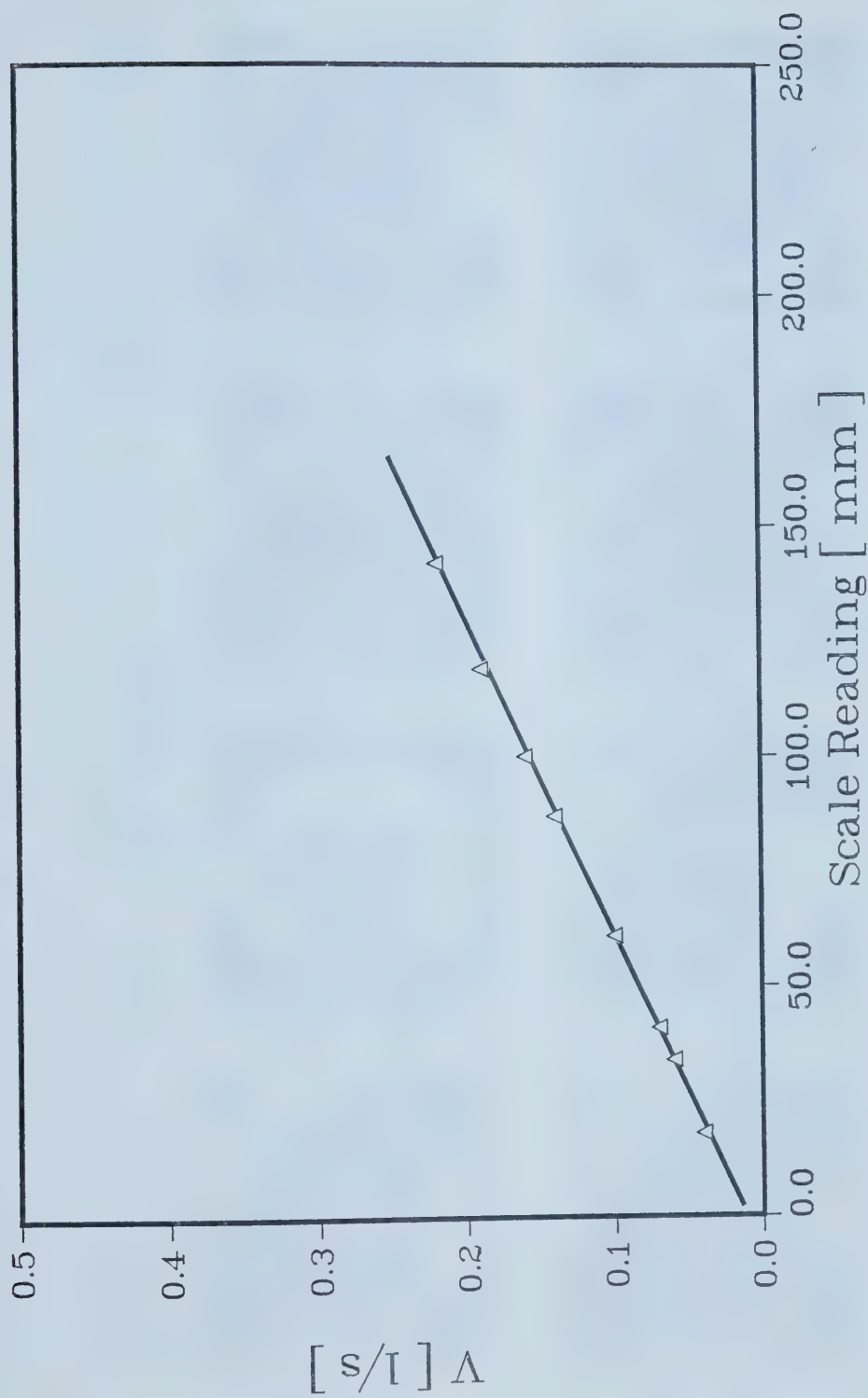


Fig. 4.3 Calibration curve for the rotameter.

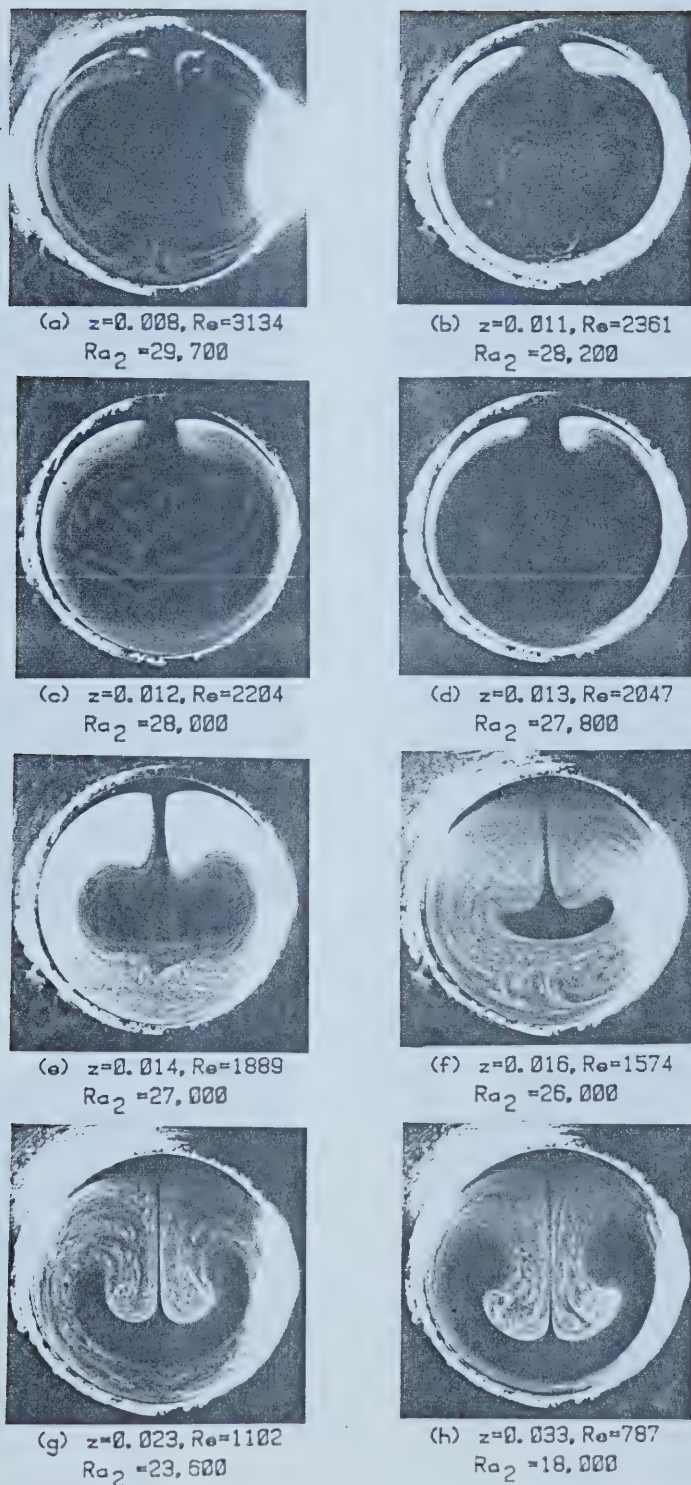
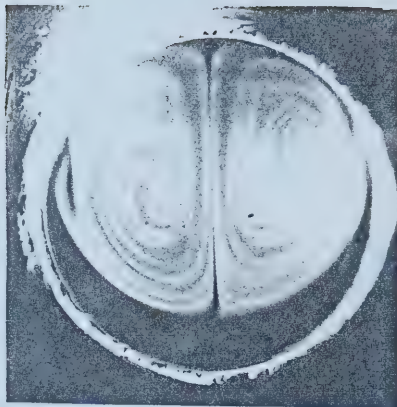


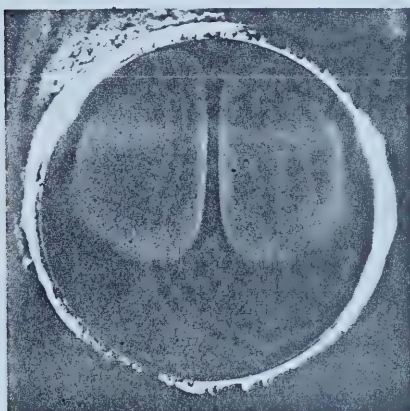
Fig. 4.4 Secondary flow patterns in a heated horizontal tube for $T_w=56^\circ\text{C}$ and $Ra_1=3.65 \times 10^4$.



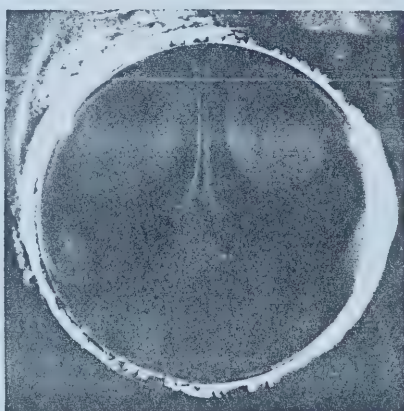
(i) $z=0.054$, $Re=472$
 $Ra_2=13,900$



(j) $z=0.082$, $Re=314$
 $Ra_2=11,100$



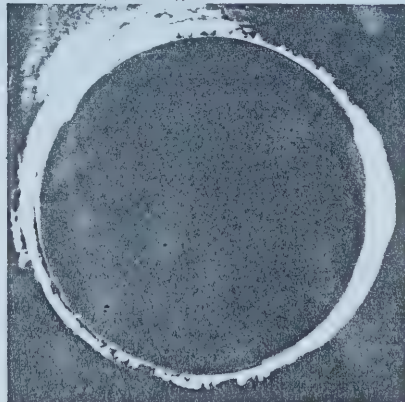
(k) $z=0.163$, $Re=157$
 $Ra_2=8,640$



(l) $z=0.299$, $Re=86$
 $Ra_2=9,620$



(m) $z=0.359$, $Re=72$
 $Ra_2=9,460$



(n) $z=0.448$, $Re=57$
 $Ra_2=9,870$

Fig. 4.4 Continued.



(a) $z=0.448, Re=57$
 $Ra_2 = 19,200$



(b) $z=0.163, Re=157$
 $Ra_2 = 9,830$

Fig. 4.5 Secondary flow patterns and plumes from the exit of a heated horizontal tube for $T_w=65^\circ$ and $Ra_1=4.37 \times 10^4$.

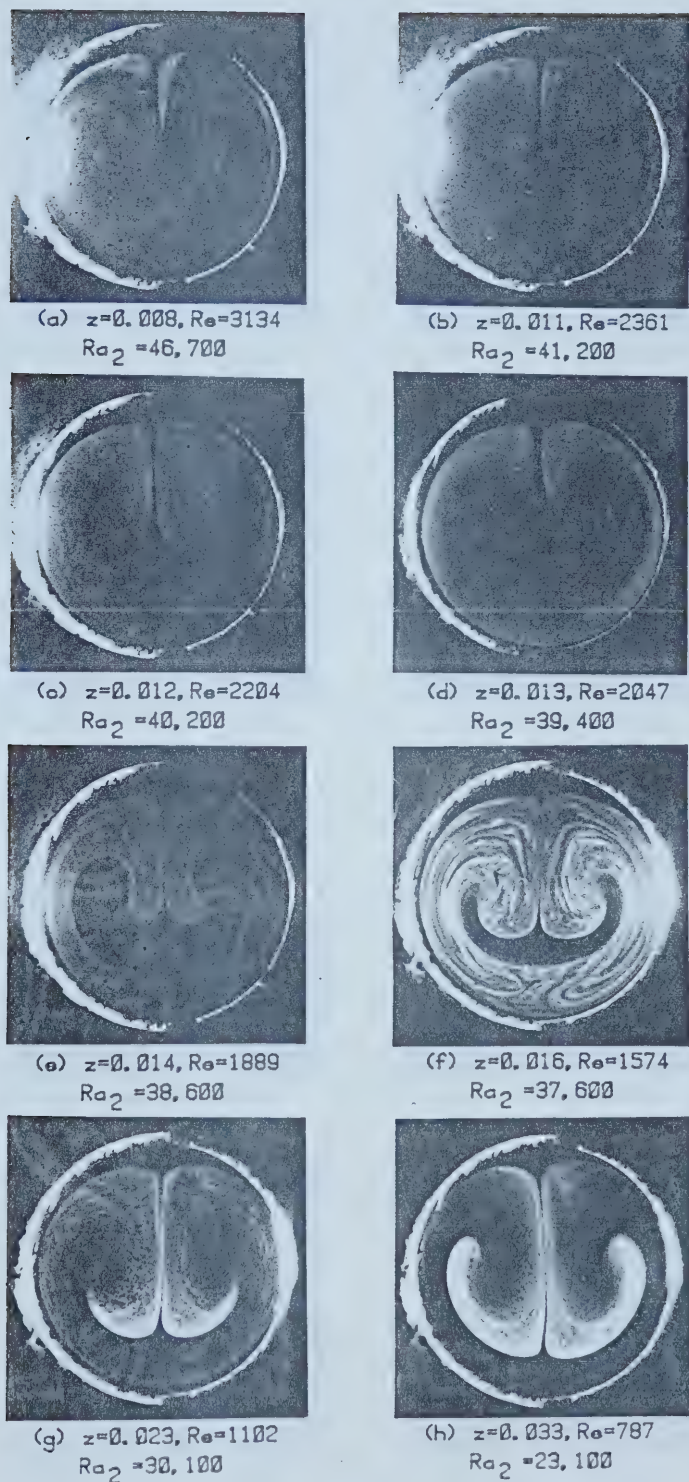


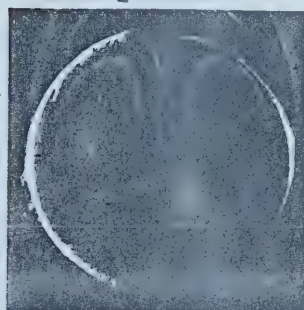
Fig. 4.6 Secondary flow patterns in a heated horizontal tube for $T_w=75^\circ\text{C}$ and $Ra_1=5.22 \times 10^4$.



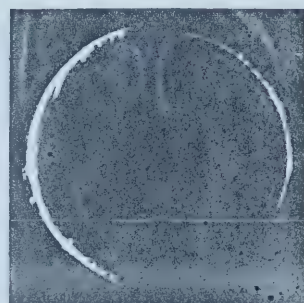
(i) $z=0.054$, $Re=472$
 $Ra_2 = 16,700$



(j) $z=0.082$, $Re=314$
 $Ra_2 = 12,900$



(k) $z=0.163$, $Re=157$
 $Ra_2 = 12,100$



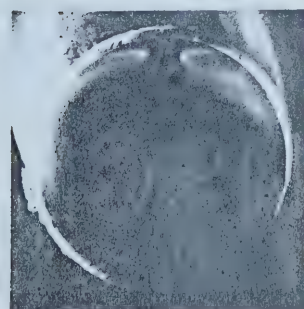
(l) $z=0.299$, $Re=86$
 $Ra_2 = 12,500$



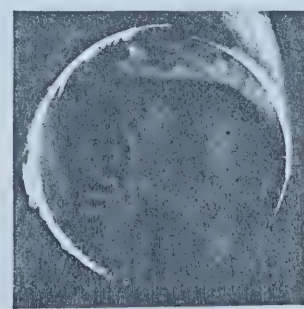
(m) $z=0.359$, $Re=72$
 $Ra_2 = 15,900$



(n) $z=0.448$, $Re=57$
 $Ra_2 = 15,600$



(o) $z=0.597$, $Re=43$
 $Ra_2 = 15,100$



(p) $z=1.833$, $Re=14$
 $Ra_2 = 14,800$

Fig. 4.6 Continued.

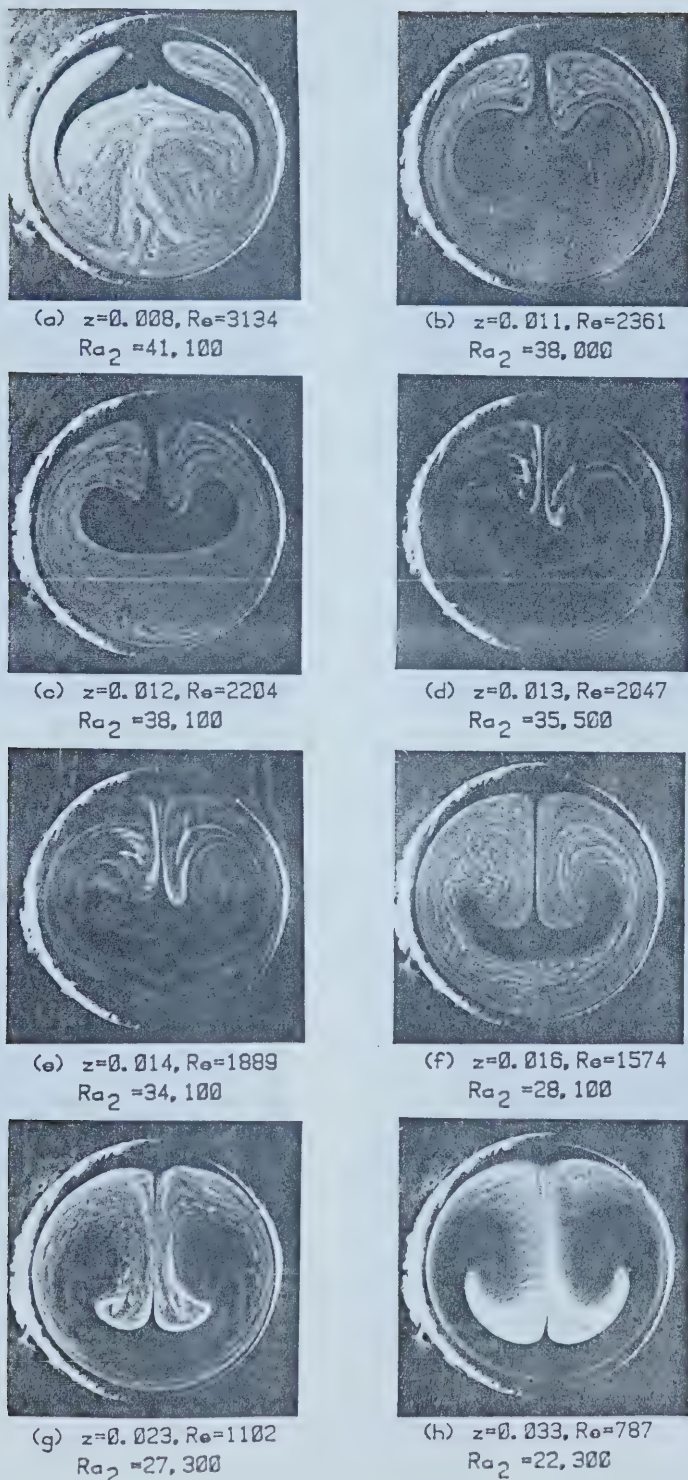
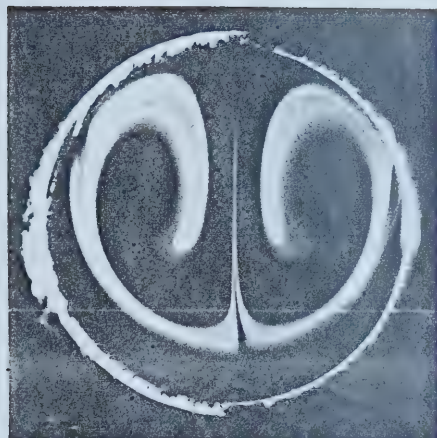
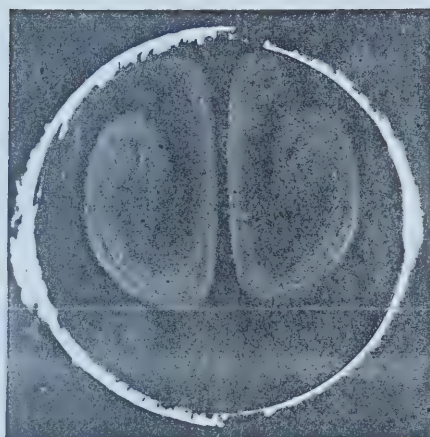


Fig. 4.7 Secondary flow patterns in a heated horizontal tube for $T_w=93^\circ\text{C}$ and $Ra_1=6.30 \times 10^4$.



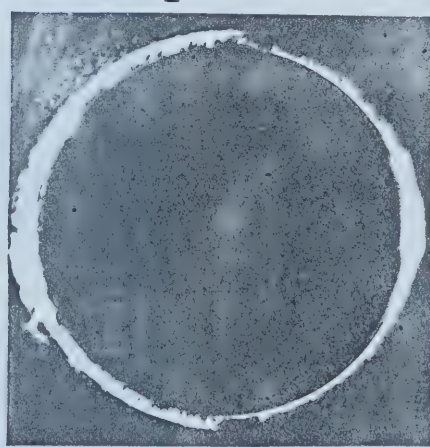
(i) $z=0.054, Re=472$
 $Ra_2 = 17,000$



(j) $z=0.082, Re=314$
 $Ra_2 = 13,500$



(k) $z=0.163, Re=157$
 $Ra_2 = 12,300$



(l) $z=0.299, Re=86$
 $Ra_2 = 14,100$

Fig. 4.7 Continued.

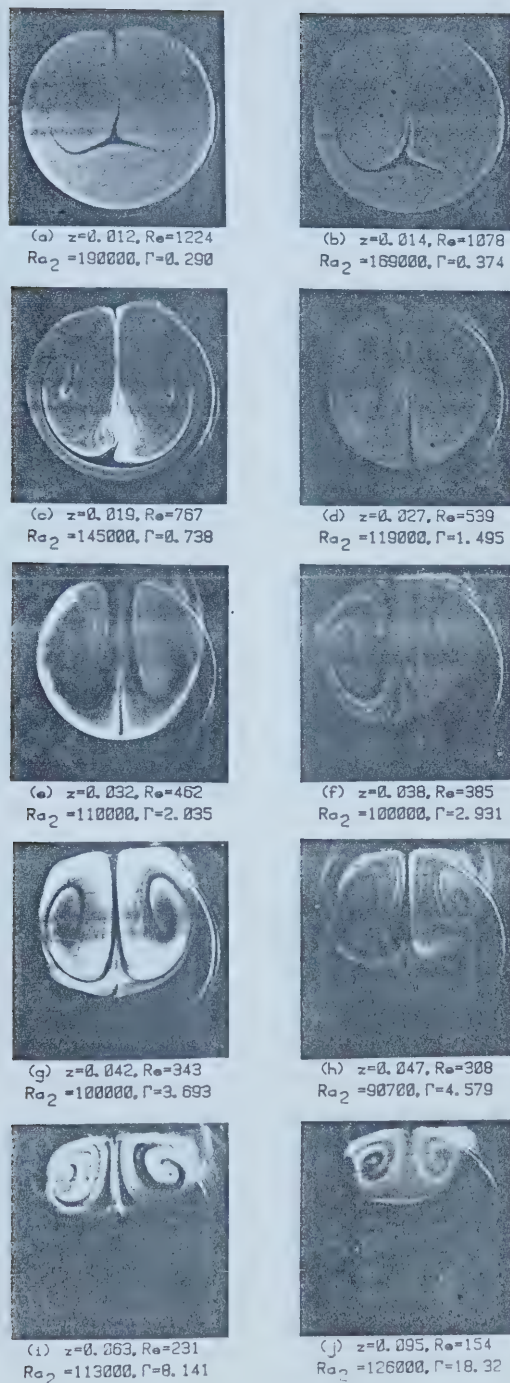


Fig. 4.8 Secondary flow patterns in a heated horizontal tubes for $T_w=56^\circ\text{C}$, $L=0.53\text{m}$ and $Ra_1=3.08 \times 10^5$.

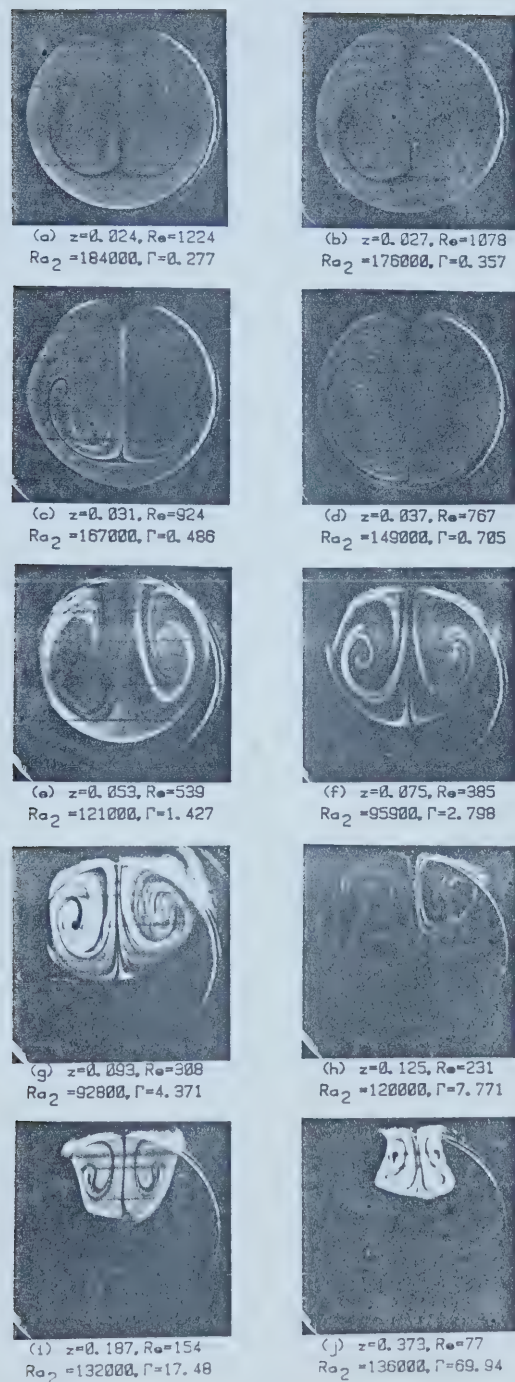


Fig. 4.9

Secondary flow patterns in a heated horizontal tubes for $T_w=56^\circ\text{C}$, $L=1.04\text{m}$ and $Ra_1=2.94 \times 10^5$.

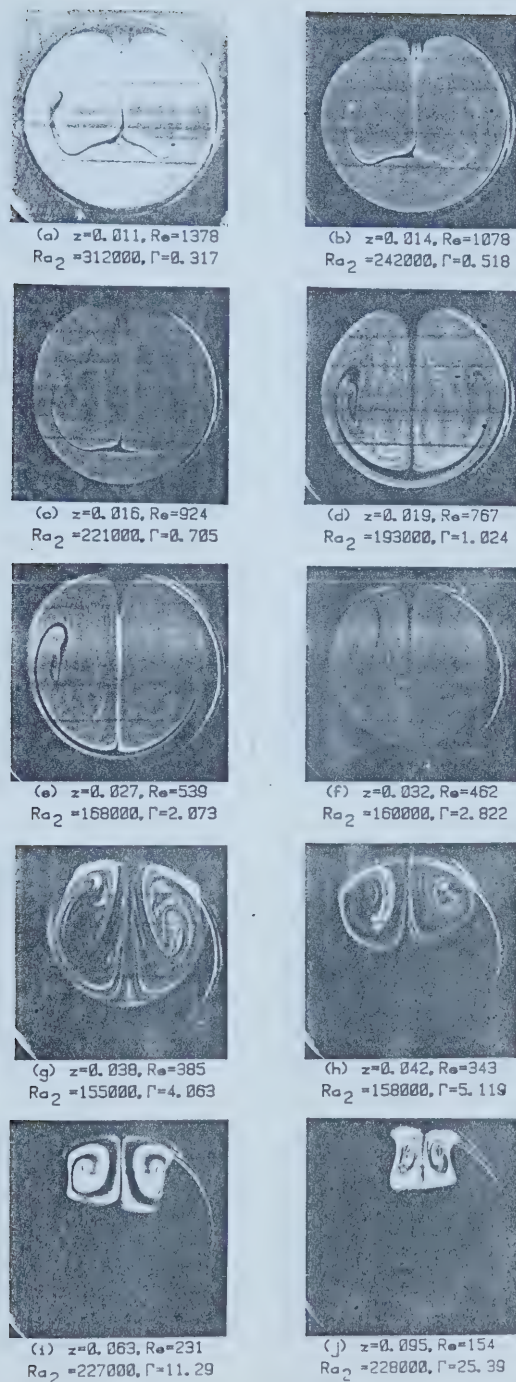


Fig. 4.10 Secondary flow patterns in a heated horizontal tubes for $T_w=75^\circ\text{C}$, $L=0.53\text{m}$ and $Ra_1=4.27 \times 10^5$.

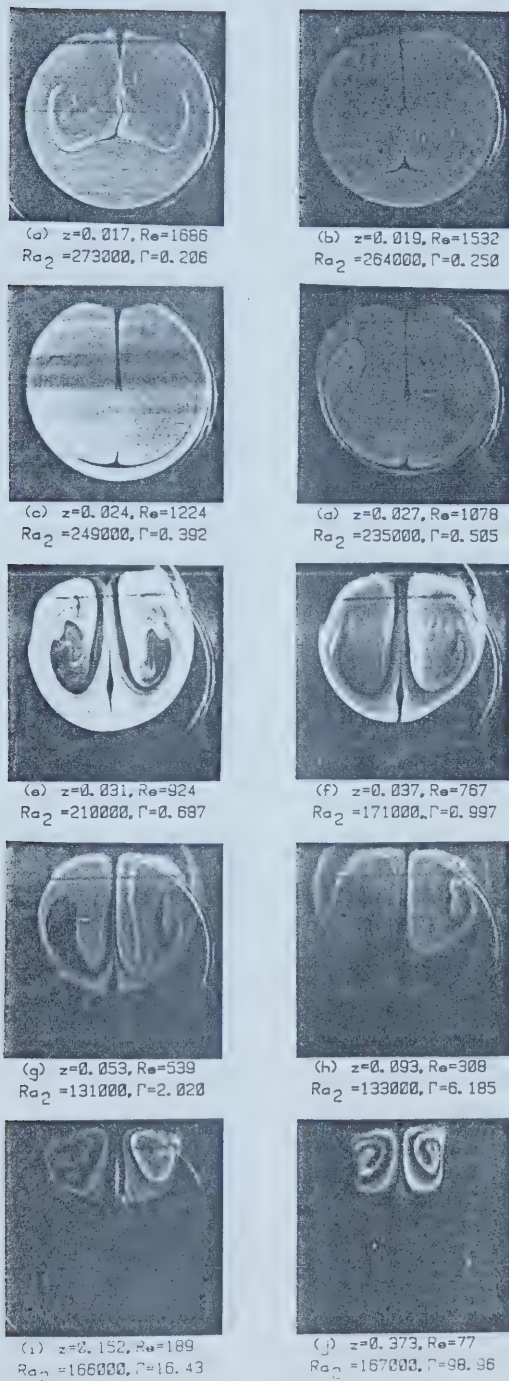


Fig. 4.11 Secondary flow patterns in a heated horizontal tubes for $T_w=74^\circ\text{C}$, $L=1.04\text{m}$ and $Ra_1=4.16 \times 10^5$.

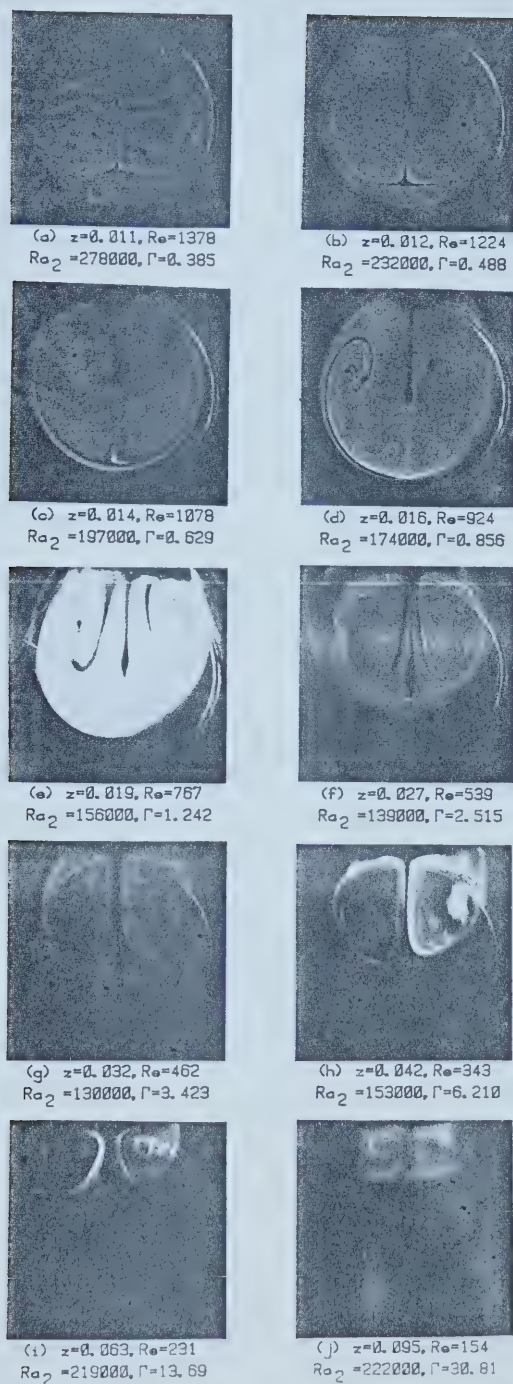


Fig. 4.12 Secondary flow patterns in a heated horizontal tubes for $T_w=94^{\circ}\text{C}$, $L=0.53\text{m}$ and $Ra_1=5.18 \times 10^5$.

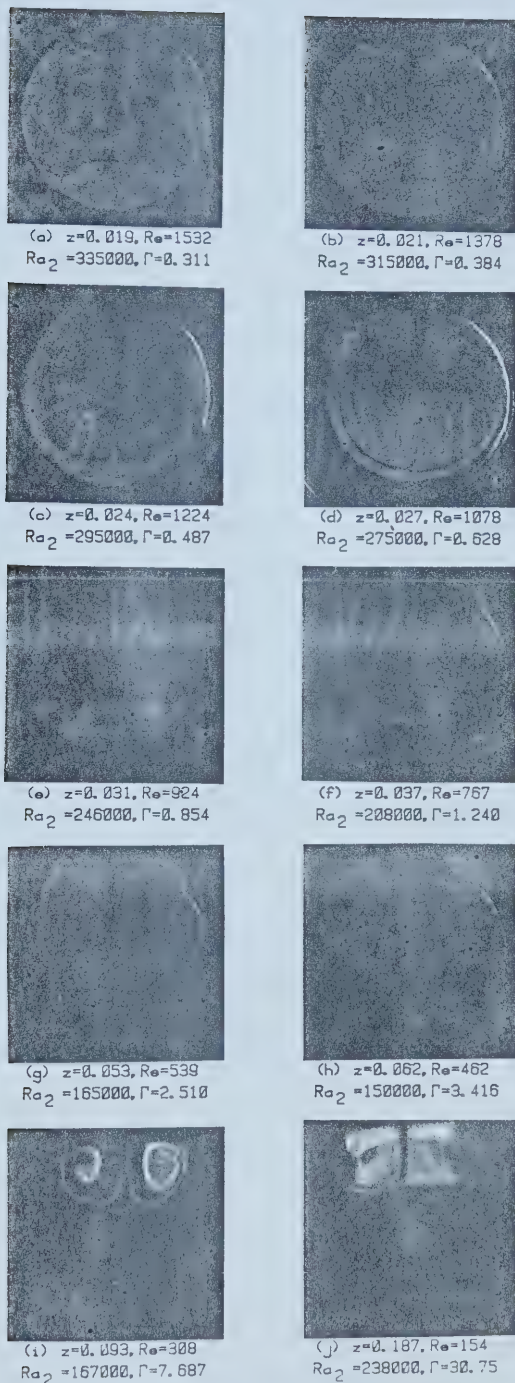


Fig. 4.13 Secondary flow patterns in a heated horizontal tubes for $T_w=94^{\circ}\text{C}$, $L=1.04\text{m}$ and $Ra_1=5.17 \times 10^5$.

5. Transient Secondary Flow in the Thermal Entrance Region of an Isothermally Heated Horizontal Pipe

Summary

The transient secondary flow patterns in the thermal entrance region of an isothermally heated straight horizontal tube are presented. Visualization was made possible by injecting smoke into air. Photographs were taken at the exit of an isothermally heated tube ($d=5.10\text{cm}$, $l=1.038\text{m}$ and $T_w=75^\circ$ and 94°C). Experimental parameters are dimensionless axial distance $z=3.10 \times 10^{-2}$ – 5.3×10^{-2} , Reynolds number $Re=924$ – 539 , Rayleigh number $Ra_1=1.17 \times 10^5$ – 4.88×10^5 and time from the start of heating $\tau=0$ – 283 sec. In the case of transient state, such interesting phenomena as the development and fluctuation of the secondary flow patterns are considered to be helpful in understanding the heat transfer mechanism of this problem.

5.1 Introduction

In Chapter 4, we have shown the developing secondary flow patterns in the thermal entrance region of an isothermally heated horizontal tube for the steady state case. One may wonder how the secondary flow would develop when the horizontal tube is being heated or cooled. Numerical and experimental solutions of the transient free convection case in a horizontal tube have previously been studied and literature can be found in[1-4]. However, the literature on transient free and forced convection in an isothermally heated tube cannot be readily found. Flow visualization study may be a viable means to understand such a complicated heat transfer mechanism. Thus, the objective of this chapter is to investigate the secondary flow patterns for the transient free and forced convection in an isothermally heated horizontal tube.

5.2 Experimental Parameters

The experimental parameters for the transient state in an isothermally heated horizontal tube are time from the start of heating τ , Reynolds number Re , Rayleigh number $Ra_1 = g\beta(T_w - T_0)d^3/\nu\kappa$ and dimensionless axial distance $z = (l/d)/RePr$. For this experiment, the pipe dimensions and ranges of experimental parameters are $d=5.10\text{cm}$, $l=1.038\text{m}$, $T_w=75-95^\circ\text{C}$, $Re=539-924$, $Ra_1=1.17 \times 10^5 - 4.88 \times 10^5$ and the ratio of Grashof number to the square of Reynolds number $\Gamma=0.193-2.369$.

5.3 Experimental Apparatus and Procedure

The experimental apparatus and procedure for the transient state in a heated horizontal tube are similar to those described in Chapter 4 with some exceptions. A schematic diagram of experimental apparatus can be seen in Fig. 4.1. More details can be found in Chapter 4 or [5].

For the transient state experiment, only the longest heating length ($l=1.038\text{m}$) was used in order to provide the maximum free convection effect. Experiments were carried out at three Reynolds numbers $Re=539$, 737 and 924 with two different heating fluid temperatures $T_s=75^\circ$ and 95°C . Two Hewlett Packard X-Y plotters were installed to measure the instantaneous water inlet and outlet temperatures. The time (τ) represents the time period of hot heating fluid being passed through the inlet at a high speed. Typical water inlet and outlet temperatures versus time is shown in Fig. 5.1.

5.4 Results and Discussion

In this section, we are interested in the formation of secondary flow patterns in the thermally developing regime. The heat conduction through the copper pipe wall thickness is so fast that the time required for conduction is ignored. Flow at steady state has been investigated in the previous chapters and will not be repeated here.

5.4.1 Photographs for $z=0.031$, $Re=924$, Figs. 5.2 and 5.3

Figs. 5.2 and 5.3 show a series of instantaneous secondary flow patterns for heating fluid temperatures $T_s=75^\circ$ and 95°C respectively. Photographs are arranged in the order of increasing time τ with Rayleigh number Ra , and Γ shown for reference. At $\tau=10\text{s.}$ in Fig. 5.2(a), secondary flow separation is clearly seen at the top of the tube. As τ increases, the fluid near the wall is lighter because of the heating effect ;and as a result, the upward fluid flow along the wall occurs. Due to the symmetry about the vertical axis and continuity, a downward fluid flow along the vertical centreline occurs. Thus, a pair of counter-rotating vortices is then formed. As shown in Fig. 5.2(c) at $\tau=20\text{s.}$, a layer of heated air has occurred at the lower part of the tube. However, it takes 8 seconds longer ($\tau=28\text{s.}$ in Fig. 5.3(d)) for the heated layer to be formed when the wall temperature is increased. A comparison of Figs. 5.2 and 5.3 also shows that a higher heating rate at the wall would cause more distortion of the secondary flow development as shown by the distortion at the centreline of the vertical axis. See Fig. 5.2(f) to Fig. 5.3(d) and Fig. 5.2(h) to Fig. 5.3(g). At $z=0.031$ in this case, it takes about $\tau=25$ sec. to develop a two-vortex secondary flow pattern. The two vortices are fairly symmetric except along the line separating the left- and right-hand regions. Such dividing line is observed to be distorted for $\tau\leq 50\text{s.}$ in Fig. 5.2. Symmetry occurs when there is more convection effect

(τ is large enough). It is also shown that the downward secondary flow in the core region cannot penetrate all the way to the bottom wall as can be seen in all photographs with $z=0.031$. The heated layer is still observed for all transient cases. One also notices that the thickness of the heated layer decreases as τ increases. Fig. 5.3(j) shows that the heated layer will disappear at $\tau=283s$. for the steady state. Generally speaking, these flow patterns reveal that for a constant flow rate, τ is the prime governing parameter to the secondary flow pattern development. In other words, secondary flow patterns are not very sensitive to the Rayleigh number, especially in the range of Rayleigh numbers discussed here. See Figs. 5.2(j) and 5.3(d) both at $Ra_1 \approx 3.9 \times 10^5$ for illustration. It is of practical interest to note that the time (τ) in transient secondary flow development is qualitatively equivalent to the axial distance (z) in the steady secondary flow development. In the steady state, a thin crescent corresponding to the dark part near the lower half of the tube occurs at $z=0.033[5]$. In the transient state, such lifting effect is not clearly defined especially at this entrance region. One concludes that the amount of secondary flow pattern being lifted up when the air temperature at the lower half of the tube approaches the wall temperature will be governed by the dimensionless axial distance z , Rayleigh number and the time τ . As τ increases, the secondary flow patterns will eventually become steady for a fixed z .

Finally, our result indicates little about the eyes of vortices due to the fact that most of the time only certain streaklines can be clearly seen.

5.4.2 Photographs for $z=0.037$, $Re=767$, Figs. 5.4 and 5.5

Figs. 5.4 and 5.5 show a series of instantaneous secondary flow patterns for $T_s=75^\circ$ and 95°C respectively. Since the heated test tube length is fixed, it is noted that z increases as Reynolds number decreases. Thus one is able to investigate flow patterns at different dimensionless axial distance z by changing the flow rate. The flow patterns are similar to the ones in Figs. 5.2 and 5.3 except that the kidney-shaped pattern is enlarged. The buoyancy force is much stronger as is evident by the larger sizes of the secondary flow patterns in Fig. 5.4(g) and 5.5(g) at $\tau=40\text{s.}$ and 48s. respectively. For a large z , a heated layer is clearly seen at $\tau=18\text{s.}$ in Fig. 5.5(c), but it has disappeared at $\tau=50\text{s.}$ in Fig. 5.4(h). In comparison, the size of the small crescent region (dark part) at the lower half of tube has significantly increased for higher wall temperature. See Fig. 5.5(j) at $\tau=123\text{s.}$ and Fig. 5.2(j) at $\tau=120\text{s.}$ for illustration. Figs. 5.4 and 5.5 reveal that higher heating rate at the wall would only cause some abrupt change to the shape of the vortices. The development of a secondary flow pattern seems to depend primarily on the τ and z . Due to the larger z and consequently more buoyancy force effect, it takes only 18 sec. to form a two-vortex

cell. However, it also takes longer to stabilize the flow patterns when compared with flow patterns at $z=0.031$ in Figs. 5.2 and 5.3.

5.4.3 Photographs for $z=0.053$, $Re=539$, Figs. 5.6 and 5.7

Figs. 5.6 and 5.7 show the secondary flow patterns for $T_s=75^\circ$ and 95° respectively. At $\tau=15s.$ in Fig. 5.6(b), a downward fluid flow along the vertical centreline is clearly seen. However, such a downward stream is not strong enough to penetrate all the way down to distinguish the left- and right-hand regions along the vertical axis of the tube. Such a dividing line is clearly seen for $\tau<90s.$, but is distorted. For $\tau\geq 90s.$, secondary flow patterns become fairly symmetric. At higher wall temperature, it takes a longer time for the flow patterns to become symmetric along the vertical axis. See Fig. 5.6(i) and Fig. 5.7(g) at $\tau=75$ sec. and Fig. 5.6(j) and Fig. 5.7(h) at $\tau=90$ sec. for typical comparisons. Due to the larger free convection effect shown by the large z value, the time required to form the two-vortex secondary flow pattern is greatly reduced (τ is about 15 sec.) in this experiment as compared Fig. 5.7(c) to Fig. 5.5(c). One notes that the thermal entrance length based on the classical Graetz solution in steady state is $z=0.05$. Also, our previous steady state results have revealed that a crescent-shaped region of secondary flow is being lifted up at $z=0.053$ as the air temperature near the lower tube section approaches the wall temperature. However

this experiment shows that such lifting effect requires some time to develop during the transient state. At such large z , a thin crescent (dark part) has already appeared even at $\tau=10\text{s}$. in Fig. 5.7(b). As τ increases, this small crescent region will grow larger. Unlike the steady state case, the growth of this crescent region has a limit which is set by the thermal entrance region (ie, z). Comparing Fig. 5.6 to Fig. 5.4, a larger crescent region can be seen for the same τ due to the large z in Fig. 5.6. Secondary flow patterns are very complicated for $\tau \leq 55\text{s}$. probably because of the strong lifting effect in the early transient state. The buoyancy effect is believed to still exist at $\tau=90\text{s}$. for $z=0.053$.

5.5 Concluding Remarks

The transient secondary flow patterns in the thermal entrance region of an isothermally heated horizontal tube are presented for the transient heating period $\tau=0-283\text{ sec}$. at two heating fluid temperatures ($T_w=75^\circ$ and 94°C) and for three different flow rates ($Re=539, 767$ and 924).

It is noted that the time τ in transient secondary flow development is qualitatively equivalent to the axial distance z in the steady secondary flow development. However, unlike the steady state case, the ascending secondary flow region in the transient state case for large time values will be dependent on both the axial distance z and the time τ . The primary parameters for secondary flow

development in transient state are the axial distance z , Rayleigh number Ra_1 , and the time τ . Secondary flow development does not seem to be very sensitive to Rayleigh number Ra_1 . However, the Rayleigh number does appear to affect the shape of vortices to be formed. In the transient state case, the line separating left- and right-hand regions along the vertical axis is distorted. Higher heating rate on the wall would cause more distortion to the line separating different regions.

This investigation provides some insight into the transient secondary flow patterns induced by the buoyancy force in an isothermally heated horizontal pipe.

5.6 References

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4. Hauf, W. and Grigull, U., "Instationarer Wärmeübergang durch freie Konvektion in horizontalen zylindrischen Behaltern", *Heat Transfer* 1970, 4, Paper NC1.3, Elsevier Pub., Amsterdam.
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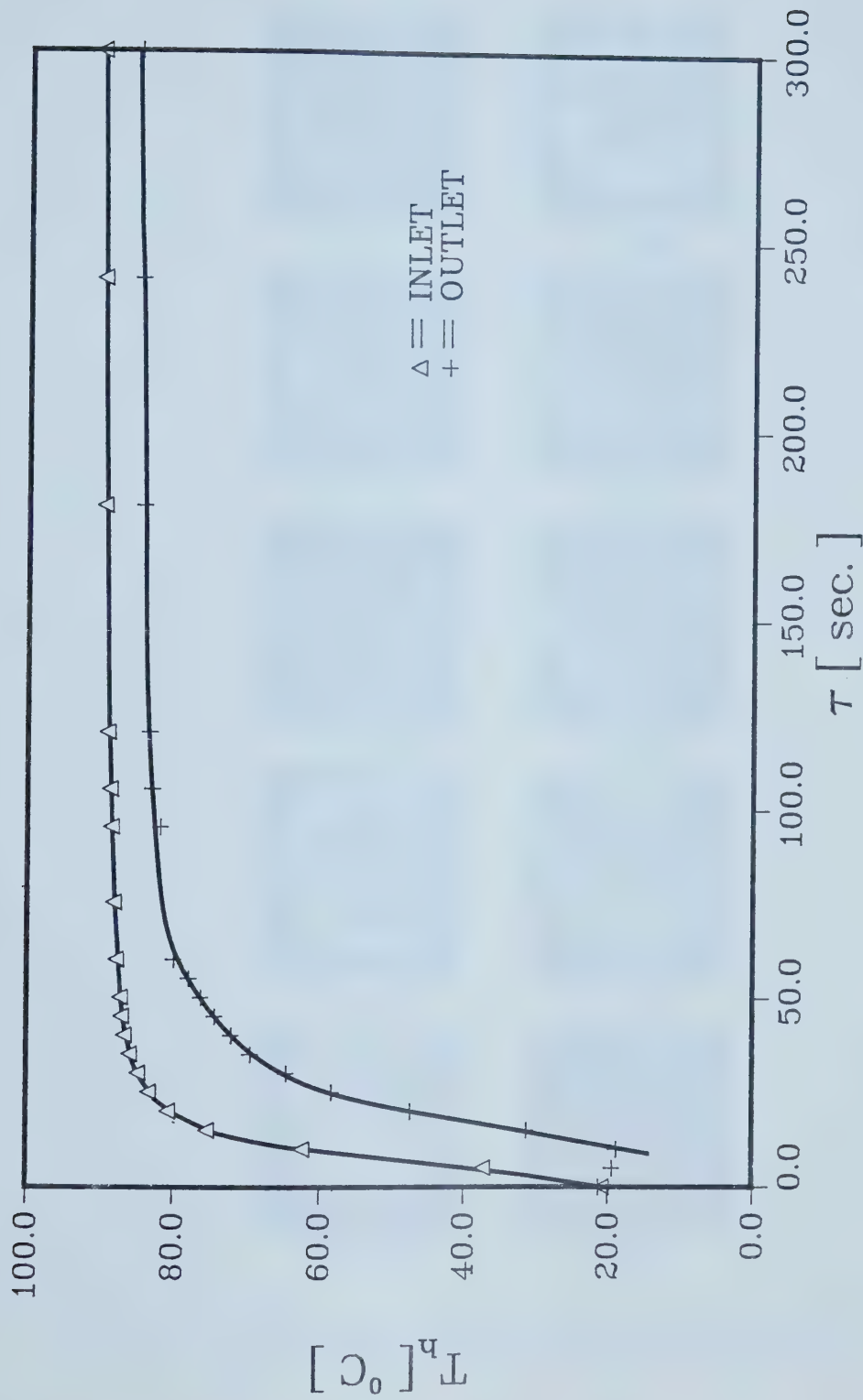


Fig. 5.1 Typical heating fluid temperatures vs. time for $T_s \approx 95^\circ\text{C}$.

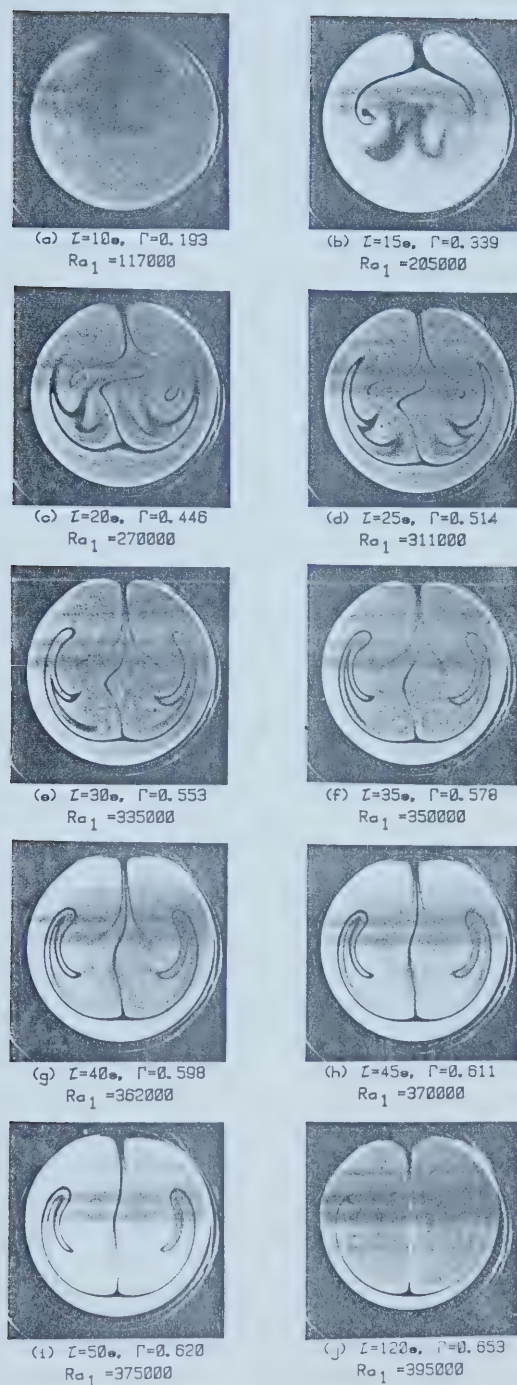


Fig. 5.2 Secondary flow patterns in a heated horizontal tube for $T_s \approx 75^\circ\text{C}$, $z=0.031$ and $Re=924$.

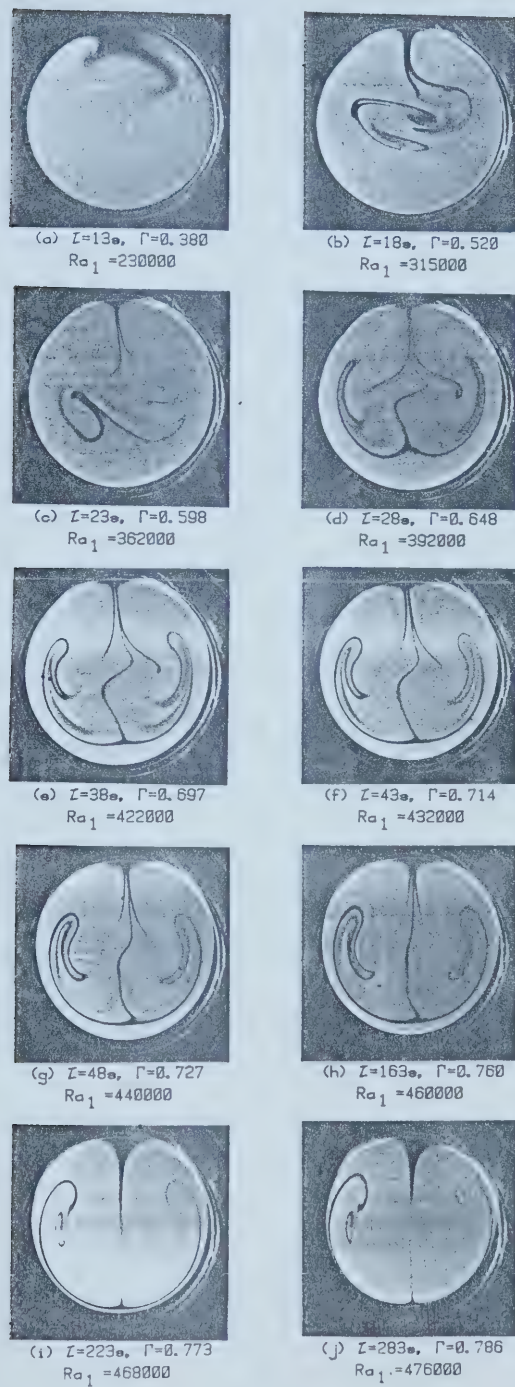


Fig. 5.3 Secondary flow patterns in a heated horizontal tube for $T_s \approx 95^\circ\text{C}$, $z=0.031$ and $Re=924$.



Fig. 5.4 Secondary flow patterns in a heated horizontal tube for $T_s \approx 75^\circ\text{C}$, $z=0.037$ and $Re=767$.

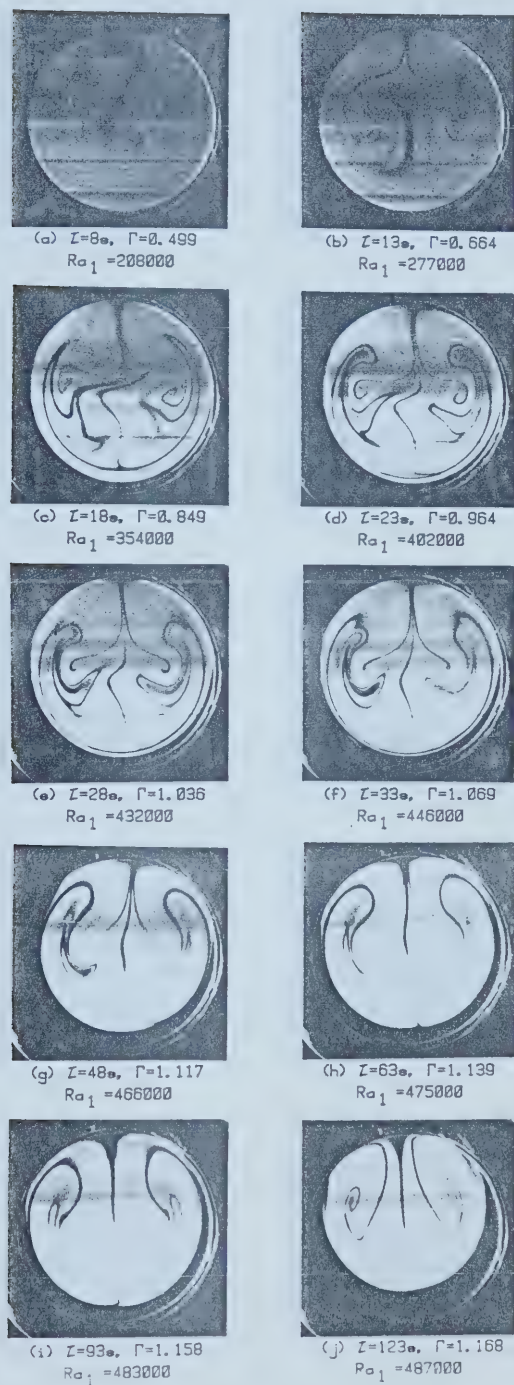


Fig. 5.5 Secondary flow patterns in a heated horizontal tube for $T_s \approx 95^\circ\text{C}$, $z=0.037$ and $Re=767$.

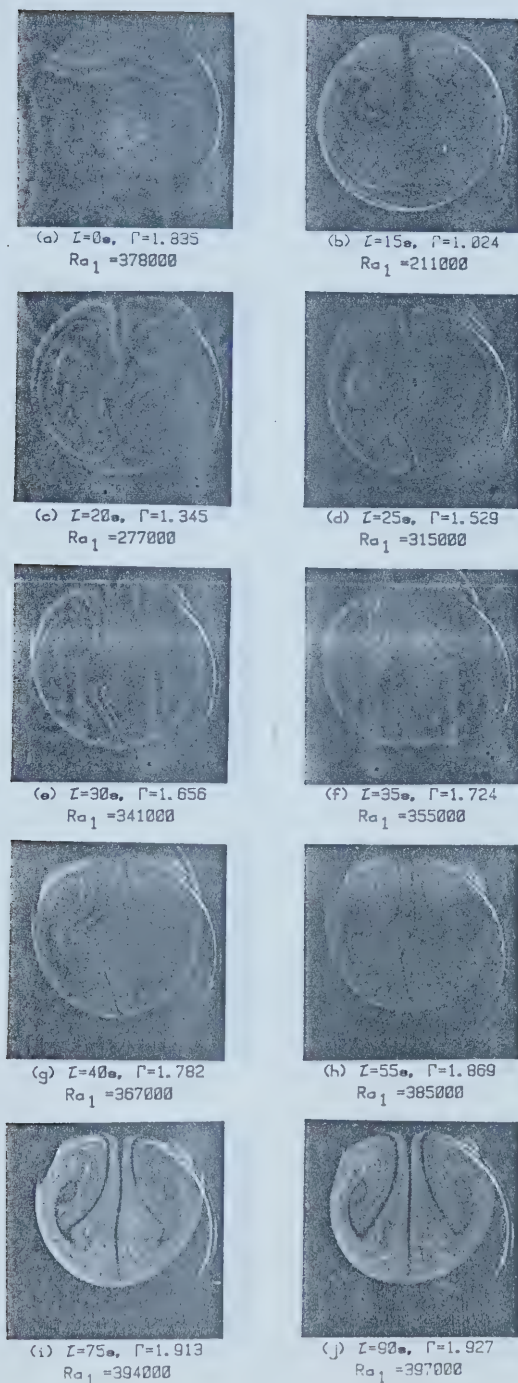


Fig. 5.6 . Secondary flow patterns in a heated horizontal tube for $T_s \approx 75^\circ\text{C}$, $z=0.053$ and $Re=539$.

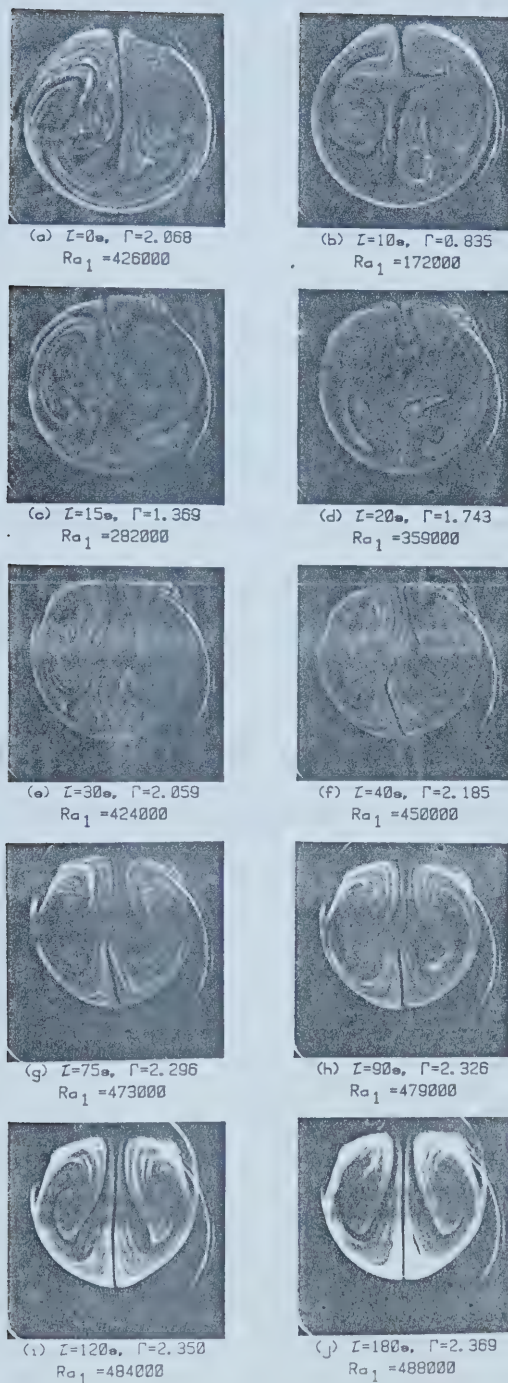


Fig. 5.7 Secondary flow patterns in a heated horizontal tube for $T_s \approx 95^\circ\text{C}$, $z=0.053$ and $Re=539$.

6. Secondary Flow in the Thermal Entrance Region of Isothermally Heated Inclined Pipes⁵

Summary

Photographs are presented for the secondary flow patterns in the thermal entrance region of isothermally heated inclined tubes. Experiments have been carried out with tube wall temperature $T_w=55-95^\circ\text{C}$, air entrance temperature $T_o\approx 25^\circ\text{C}$, inverse Graetz number $z=0.008-1.833$ ($Re=3134-14$) and inclination angles $\theta=30^\circ$, 45° and 60° from horizontal. The flow was in the upward direction. The characteristic features of the secondary flow induced by buoyancy forces for Graetz problem in inclined tubes with significant free convection effects are clearly delineated by flow visualization studies. Its decay process and subsequent disappearance in the downstream region are also studied.

⁵A version of this chapter has been accepted for publication. Cheng, K.C. and Yuen, F.P. ASME paper, 23rd ASME/AIChE/ANS National Heat Transfer Conf., Denver, Colorado, Aug. 4-7, 1985.

6.1 Introduction and Historical Observations

Since the pioneer works of Graetz[1] in 1885 and Nusselt[2] in 1910 on pure laminar forced convection in thermally heated tubes with fully developed laminar flow, the convective heat transfer problems concerning the effects of variable, temperature-dependent thermophysical properties of fluid and the thermal entrance region problems in pipes and channels of various geometrical cross-sections have been studied, both theoretically and experimentally, by many investigators because of their practical significance in applications. During the past century, a considerable number of articles and papers in this area of convective heat transfer have been published.

The importance of buoyancy effects (free convection) on thermal entrance region problems (Graetz problem) has been recognized since the beginning of this century and earlier results are well summarized by McAdams[3] and Schack[4]. Recent literature can be found in[5-9]. When free convection is significant in a laminar forced convection problem, the position of the pipe, whether horizontal or vertical; and the direction of flow, whether upward or downward, is of great significance. For the Graetz problem with significant buoyancy effects in horizontal isothermal tubes, the buoyancy effect which depends on the Rayleigh number begins first in the Leveque solution region[10] near the thermal entrance, and continues in the intermediate region before the bulk temperature approaches the constant

wall temperature for the fully developed laminar forced convection region with the well-known asymptotic Nusselt number $Nu_{\infty}=3.66$. It is of practical interest to note that when this value is reached, the tube is not very effective in convective heat transfer and thus should not be utilized in heat exchanger design. For Graetz problem, the classification of the heat transfer regime into three regions, the Leveque solution region, the intermediate region and the fully developed region, is believed to be useful in understanding the heat transfer mechanism and buoyancy effects.

It is known that the Leveque solution is analogous to to the semi-infinite solid solution for transient heat conduction problem with a uniform initial temperature, and a sudden change of the surface to a constant but different temperature. It is also of considerable interest to observe that the similarity variable for the semi-infinite solid problem is similar to the similarity variable used by Blasius in solving the Prandtl laminar boundary layer problem for a flat plate. The Graetz problem is parabolic, and for a fully developed region becomes elliptic.

In this connection, it is also observed that the development of the thermal boundary layer in the Graetz problem (1885) is somewhat similar to the Prandtl laminar boundary layer for a flat plate (1904). Presumably, Prandtl was aware of Lorenz's work[11] (1881) on thermal boundary for natural convection along a vertical flat plate but

apparently was not aware of Graetz's work (1883, 1885) on forced laminar convection in tubes. In retrospect, one also observes that some analogy exists between the semi-infinite solution for transient heat conduction problems and the Prandtl laminar boundary layer for a flat plate.

With free convective (secondary) flow superimposed on the main flow, one can visualize a pair of longitudinal vortices inside the horizontal tube. It is thus seen that the secondary flow caused by the buoyancy force near the tube wall contributes to better mixing and the local heat transfer coefficient increases. When the bulk temperature approaches the constant wall temperature, the secondary flow eventually disappears. Because of the complicated heat transfer mechanism involved, the most recent attempt[8] at a semi-analytical heat transfer correlation for laminar mixed convection in an isothermally horizontal tube using the available heat transfer data shows a deviation of about 10%. It does not appear that a similar correlation equation is currently available for inclined tubes including the practical vertical tube. The secondary flow phenomenon also occurs in an isothermally heated inclined tube, except in the vertical tube case.

For the Graetz problem with significant buoyancy effects in inclined isothermal tubes, the classical method of solution is not applicable and one must employ the numerical solution [7,9]. The flow visualization studies on developing secondary flow patterns in the thermal entrance

region of isothermally heated inclined tubes should provide some insight into the heat transfer mechanism and complement the theoretical and experimental investigations in the future. The results for the case of an isothermally heated horizontal tube are reported in [12].

The purpose of this chapter is to obtain a series of photographs on developing secondary flow patterns in the thermal entrance region of isothermally heated inclined tubes with significant free convection effects. Tubes are set up for experiments with inclination angles of $\theta=30^\circ$, 45° and 60° from horizontal with upward flow. The flow visualization was facilitated by injecting smoke into the air stream. The results can be used for future comparison with numerical solution for Graetz problem with buoyancy effects and confirming the theoretical model.

6.2 Experimental Parameters

The experimental parameters are; angle of inclination from horizontal, θ , Rayleigh numbers $Ra_1=g(\cos\theta)\beta(T_w-T_o)/\nu\kappa$ and $Ra_2=g(\cos\theta)\beta(T_w-T_c)/\nu\kappa$ and the dimensionless axial distance (inverse Graetz number) $z=(l/d)/RePr$. The range of values for experimental parameters are given in Table 6.1.

6.3 Experimental Apparatus and Procedure

A schematic diagram of the experimental apparatus is shown in Fig. 6.1. The apparatus and experimental procedure are essentially the same as those used in earlier

Table 6.1 Ranges of Experimental Parameters

Inclination Angle, θ	30°, 45°, 60°
Air Flow Rate	4.72-1033.6 cm ³ /sec
Mean Velocity	0.87-191.0 cm/sec
Reynolds number, Re	14-3134
Rayleigh Number, Ra ₁	1.92 x10 ⁴ -5.40 x10 ⁴
Rayleigh Number, Ra ₂	4.49 x10 ³ -3.62 x10 ⁴
Prandtl Number,	0.71
z	0.008-1.833
Γ	0.008-316.5

investigations[12,13]. The test section (heating length $l=46.2$ cm, tube internal diameter $d=2.63$ cm) was made from Type L copper tubing with a heating jacket. Hot water was circulated at high speed between the constant temperature bath and the test section in a counterflow direction. A helical coil was inserted in the annular space for heat transfer augmentation and the test section was insulated to prevent heat loss. The inlet and outlet bulk temperatures of the heating fluid (hot water) to the heating section were measured using 0.7mm o.d. sheathed iron-constantan thermocouples. The inlet and outlet centreline air temperatures in the test section were measured using two Type T thermocouples (dia.=0.254mm). All thermocouples were initially calibrated using a quartz thermometer (HP-2807A), with a maximum error of $\pm 0.3^\circ\text{C}$. Since the inlet

and outlet hot water temperatures were found to be nearly equal under steady state conditions, the average value was taken as the average tube wall temperature T_w .

In order to ensure that the flow is fully developed when it enters the heating test section, a long entrance is provided prior to the test section. The length of this entrance is determined by the correlation ($l/d > 0.05Re$). Room air from a compressor was used. The air flow rate was measured using a Meriam laminar flow element (Model 50MW20-1) which employs a Dwyer inclined manometer. The experimental error is estimated to be $\pm 1.7\%$. Smoke was generated by burning a bundle of Chinese straw paper sticks inside a copper smoke generator tube (see Fig. 6.1). The test section was preceded by 1.9m straight copper tubing, 5.0m curved Tygon tubing and then 1.3m straight copper tubing to a settling chamber. The test section was supported by a tilting platform.

The secondary flow patterns at the exit of the isothermally heated section were photographed from a direction facing the pipe exit. Illumination was provided by a thin plane of light from a 500W projector. The plane of light was obtained by putting two razors edges close together in a slide mounting frame. Pictures were taken in dark surroundings where only the smoke-filled flow in a plane transverse to the pipe axis was highlighted by the illumination. The heated air at the pipe exit was discharged into the atmosphere. A glass plate was placed

between the camera and the exit of the test section to protect the camera lens. A Nikon FM2 single lens reflex camera with a 50mm micro lens was employed. Kodak Tri-X black and white film was used throughout the tests. The aperture used was f3.5 while the shutter speed ranged from $1/2$ - $1/8$ of a sec. For most cases, a speed of $1/2$ sec. gave satisfactory pictures.

A curved Tygon tube was used because of space limitation. The radius of curvature R_c was 8.67, 5.11 and 3.22 m for the inclination angles $\theta=30^\circ$, 45° and 60° , respectively. The inclination angle was based on the slope of the tilting platform, with an accuracy of $\pm 1^\circ$. With $Re=500$, the Dean number $K=Re(d/2R_c)^{1/2}$, Re was less than 32 for all curvature ratios and the curved tube effect can be neglected at the thermal entrance. With $Re=1574$ the Dean number is 61.3, 79.9 and 101.0 for $R_c=8.67$, 5.11 and 3.22 m, respectively. At $Re=3134$, the Dean number K is 122 for $R_c=8.67$ m. Even though the heating section was preceded by a $72d$ straight tube, some uncertainty may exist regarding the curved tube flow effect at the thermal entrance for $Re \geq 1574$. In most cases, the curved tube effect is believed to be negligible in this investigation.

The thermal entrance length effect is characterized by the inverse Graetz number $z=(l/d)/RePr$. Since the tube length l is fixed in this study, the value for z was varied by changing Reynolds number. It is also noted that the secondary flow pattern near $z=0$ cannot be obtained in this

study.

6.4 Results and Discussion

The main interest here is to study the developing secondary flow patterns as a manifestation of free convection effects on the solution of the Graetz problem. The photographic results are shown in Figs. 6.2 to 6.10 for inclination angles, $\theta=30^\circ$, 45° and 60° each with three different constant wall temperature levels and upward flow conditions. For each constant wall temperature, photographs are shown at 10 different dimensionless axial distances, z , and the Reynolds number is also indicated for reference. In interpreting the results, it is noted that at $z=0.05$ the local Nusselt number already approaches the limiting value $Nu_\infty=3.66$ with a maximum deviation of 1% based on the Graetz solution without free convection effects. The upstream region $z=0-10^{-3}$ can be regarded to be in the Leveque solution region and is outside the scope of the present study. The parameter Γ represents only the ratio of the free convection effect over the forced laminar convection near the thermal entrance $z=0$. The photographic results for the horizontal case $\theta=0^\circ$ are given in [12].

6.4.1 Photographs for $\theta=30^\circ$, Figs. 6.2 to 6.4

The developing secondary flow patterns for the case with constant wall temperature $T_w=56.8^\circ\text{C}$ are shown in Fig. 6.2 for the range $z=0.016-0.597$ and $Ra_1=3.27 \times 10^4$. At

$z=0.016$, one sees clearly a demarcation line between the left-hand and right-hand regions only in the upper part and the interaction between the two regions occurs in the lower part leading to rather complicated secondary flow patterns which show no symmetry with respect to a vertical centreline. A layer of heated air is seen to extend from the bottom along the side wall in the upward direction. Thus, the downward secondary flow in the core region cannot penetrate to the bottom wall. The rather complicated secondary flow patterns in the lower part may be due to the heating from below, a situation which is always unstable and causes disturbances in the flow field. At $z=0.023$ and 0.033 , a central demarcation line penetrates further downward and a line of symmetry between the left-hand and right-hand regions can gradually be seen in the upper part. The secondary flow patterns shown in Figs. 6.2(a), (b) and (c) are apparently new and no reported theoretical results are available for comparison.

At $z=0.041$, a fairly good symmetry is observed between the two territories and the secondary flow pattern with two counter-rotating vortices is a phenomenon well documented by other researchers. A thin crescent corresponding to the dark area near the lower half of the tube wall with gradually decreasing thickness from the bottom along the side wall is also clearly seen at $z=0.041$. In this crescent region, the air temperature is apparently uniform and approaches the wall temperature and the secondary flow

disappears completely. At $z=0.033$, a very thin crescent already appears near the bottom. Further downstream of $z=0.041$, the area of the crescent gradually increases and a pair of fairly symmetric vortices is located in the upper part. The slight deviation from symmetry for the nearly half moon shown in Fig. 6.2(h), is caused by the disturbances at the exit of the heating section where air leaves as a jet at a lower Reynolds number. It is noted that z increases as Reynolds number Re decreases.

From the good symmetry shown in Fig. 6.2(g) for vortices and crescent, one may infer that the constant wall temperature boundary condition was achieved. At $z=0.359$, a pair of vortices is contained in the circular sector region. At $z=0.597$, the secondary flow is rather weak and the two unsymmetric vortices occupy a rather small region near the top wall. Thus, the thermal entrance length for this case with $T_w=56.8^\circ$ and $Ra_1=3.27 \times 10^4$ is seen to be around $z=0.597$ as compared with the thermal entrance length $z=0.05$ based on the classical Graetz solution. It is believed that considerable computing time is required for a numerical solution to reach $z=0.597$ and this information may be useful for future numerical analysis. At $z=0.597$ ($Re=43$), one may conclude that the bulk temperature is approaching the constant wall temperature.

The appearance of a small crescent region with negligible buoyancy forces near the lower wall at a certain z and the eventual growth to the whole flow field with

further increase of z for the case with $\theta=30^\circ$ is similar to the phenomena observed for the case of a horizontal tube with $\theta=0^\circ$ and similar constant wall temperature[12].

However, the secondary flow patterns for $z \leq 0.033$ are characteristically different from those of the horizontal tube with $\theta=0^\circ$. The developing secondary flow patterns for the present problem with large Prandtl number are shown in [7] for $Ra_1=5 \times 10^4$ and 10^5 but a direct comparison with $Pr=0.71$ in this study is not possible.

The developing secondary flow patterns for $T_w=75.8^\circ\text{C}$ and $Ra_1=4.57 \times 10^4$ are shown in Fig. 6.3. The Rayleigh number Ra_2 gives a good indication of the relative strength of the buoyancy forces for the whole heating region. At $z=0.016$ and 0.023 , a resemblance of symmetry is observed only in the upper part and again it is seen that the downward secondary flow in the central core region cannot penetrate all the way to the bottom wall. At $z=0.033$, a central demarcation line is clearly seen and a small crescent already appears near the lower wall. Further downstream, the phenomenon is quite similar to that observed for the case with $T_w=56.8^\circ\text{C}$ but with good symmetry between the left-hand and right-hand regions. The weak secondary flow patterns at $z=0.448$ ($Re=57$) suggests that the thermal entrance length may decrease with the increase of Ra_1 .

The developing secondary flow patterns for $T_w=91.5^\circ\text{C}$ and $Ra_1=5.40 \times 10^4$ are presented in Fig. 6.4. At $z=0.008$, the Reynolds number $Re=3134$ is already in excess of the

critical value of 2300. However, because of the stabilizing effect due to secondary flow the entire flow field is seen to be laminar. Apparently the re-laminarization due to buoyancy forces is of considerable theoretical and practical interest for future investigation. Again, because of the heated air layer near the lower wall, a central demarcation line cannot reach the bottom wall. The two vortices are fairly symmetric. At $z=0.016$, the interaction between the two regions can be seen in the lower part. At $z=0.033$, a small crescent already appears near the lower wall and one sees two symmetric vortices. Further downstream, the gradual growth of the crescent region in the lower part and the gradual vanishing of the secondary flow in the upper part are observed.

6.4.2 Photographs for $\theta=45^\circ$, Figs. 6.5 to 6.7

The developing secondary flow patterns for $T_w=58.1^\circ\text{C}$ ($Ra_1=2.75 \times 10^4$), $T_w=77.9^\circ\text{C}$ ($Ra_1=3.85 \times 10^4$) and $T_w=91.4^\circ\text{C}$ ($Ra_1=4.40 \times 10^4$) are shown in Figs. 6.5, 6.6, and 6.7, respectively, with some identical dimensionless axial positions for easy comparison on the buoyancy forces effect. Thus, an increase in angle of inclination results in a smaller free convection effect for the same axial distance. The thermal entry effect on secondary flow pattern is qualitatively similar to that observed for $\theta=30^\circ$. In Fig. 6.5, one sees two kidney-shaped vortices near the lower wall at $z=0.016$ and the interaction between the two regions in

the lower part persists to the downstream position $z=0.033$. At $z=0.041$, one sees a fluid layer separates a thin crescent and a pair of vortices in the upper part.

In Fig. 6.6 with $T_w=77.9^\circ\text{C}$, a heated layer near the lower wall can be seen clearly at $z=0.016$ and the secondary flow pattern is seen to be rather complicated. The flow patterns at $z=0.016$ and 0.023 suggest that one cannot assume symmetry between the left-hand and right-hand regions in numerical solution. A demarcation line is clear in each photograph. In Fig. 6.7 with $T_w=91.4^\circ\text{C}$, the secondary flow is seen to be rather weak at $z=0.597$ but a remnant still persists at $z=1.833$ ($Re=14$). At $z=0.016$ and 0.023 , the demarcation line from the top cannot penetrate to the bottom due to the heated air layer near the lower wall.

6.4.3 Photographs for $\theta=60^\circ$, Figs. 6.8 to 6.10

The photographic results for $T_w=57.4^\circ\text{C}$ ($Ra_1=1.92 \times 10^4$) $T_w=77.9^\circ\text{C}$ ($Ra_1=2.72 \times 10^4$) and 91.3°C ($Ra_1=3.11 \times 10^4$) are presented in Figs. 6.8, 6.9 and 6.10, respectively. The effects of the buoyancy forces and entrance distance on the formation of secondary flow patterns for inclination angle $\theta=60^\circ$ are qualitatively similar to those for $\theta=30^\circ$ and 45° . At $z=0.016$, a heated air layer near the lower wall exists and several vortices can be observed. At $T_w=91.3^\circ\text{C}$, the symmetry of secondary flow is already seen at $z=0.033$. For $T_w=77.9^\circ\text{C}$, the symmetry of the two regions is seen in the range $z=0.016-0.163$. For $T_w=57.4^\circ\text{C}$, the symmetry begins at

$z=0.054$. After the establishment of a symmetric secondary flow, a crescent appears near the lower wall and the buoyancy effect gradually decreases further downstream. Judging from the area of the secondary flow region, the buoyancy effect can be practically neglected for $T_w=77.9^\circ\text{C}$ and 91.3°C downstream of $z=0.597$. At $T_w=57.4^\circ\text{C}$, the thermal entrance length appears to be longer than those at $T_w=77.9^\circ\text{C}$ and 91.3°C . In studying the effects of inclination angle and thermal entrance on secondary flow pattern, it is noted that the magnitude of Ra_2 represents the intensity of the secondary flow.

The presented photographic results enable one to study the wall temperature effects on the secondary flow pattern for a particular inclination angle and the effect of the inclination angle on the secondary flow pattern at a particular wall temperature qualitatively. However, within the scope of this investigation, a definite change in secondary flow pattern cannot be ascertained. For given constant wall temperature and Reynolds number, the secondary flow intensity is strongest for the horizontal tube but the secondary flow disappears in the vertical tube. With the present flow visualization method, secondary flow patterns for $z<0.008$ cannot readily be obtained because of the smoke diffusion at higher velocities (Reynolds number <3134). The secondary flow patterns are rather complicated in the range of $0.008 \leq z < 0.033$ and one can see the interaction between the left-hand and right-hand regions in the lower part. The

appearance of a crescent region (dark area) near the lower wall at a certain value of z is of particular interest. When the crescent region appears, one sees only a pair of counter-rotating vortices near the top wall further downstream. When the secondary flow disappears (or the influence of buoyancy forces vanishes), the temperature field becomes fully developed and the limiting value $Nu_{\infty}=3.66$ is approached. The corresponding value of z represents a thermal entrance length. Within the scope of this investigation, the thermal entrance lengths with buoyancy effects are in the range of $z=0.597-1.833$ which is considerably longer than $z=0.05$ from the classical Graetz solution.

6.5 Concluding Remarks

The secondary flow patterns induced by buoyancy forces in the thermal entrance region of isothermally heated inclined tubes are presented for the dimensionless downstream distances from the thermal entrance in the range of $z=0.008-1.833$ ($Re=3134-14$) at three constant wall temperature levels in the ranging from $T_w=55-95^{\circ}\text{C}$. Entrance air temperature T_o was maintained at about 25°C for three inclination angles, $\theta=30^{\circ}, 45^{\circ}$ and 60° from horizontal, with upward air flow.

The secondary flow patterns are rather complicated ranging from $z=0.008$ to 0.023 showing the interaction between the left-hand and right-hand regions in the lower

part of the flow field. After the appearance of a crescent region near the lower wall at a certain downstream distance $z \geq 0.033$ depending on wall temperature and inclination angle, only a pair of counter-rotating vortices remains in the upper part, and eventually vanishes when a fully developed temperature field without the buoyancy effect is reached.

With the influence of the buoyancy forces in the form of secondary flow, it was confirmed that a flow field remains laminar at $Re=3134$. The cross-sectional view at such a high Reynolds number is of special interest. The region $z < 0.008$ remains to be investigated in future studies. This investigation together with the earlier one [12] should be useful for future numerical and experimental studies.

It is seen that for convective heat transfer problems with secondary flow, flow visualization experiments provide considerable physical insight into the heat transfer mechanism. Thus, flow visualization experiments should be used to confirm theoretical modelling.

6.6 References

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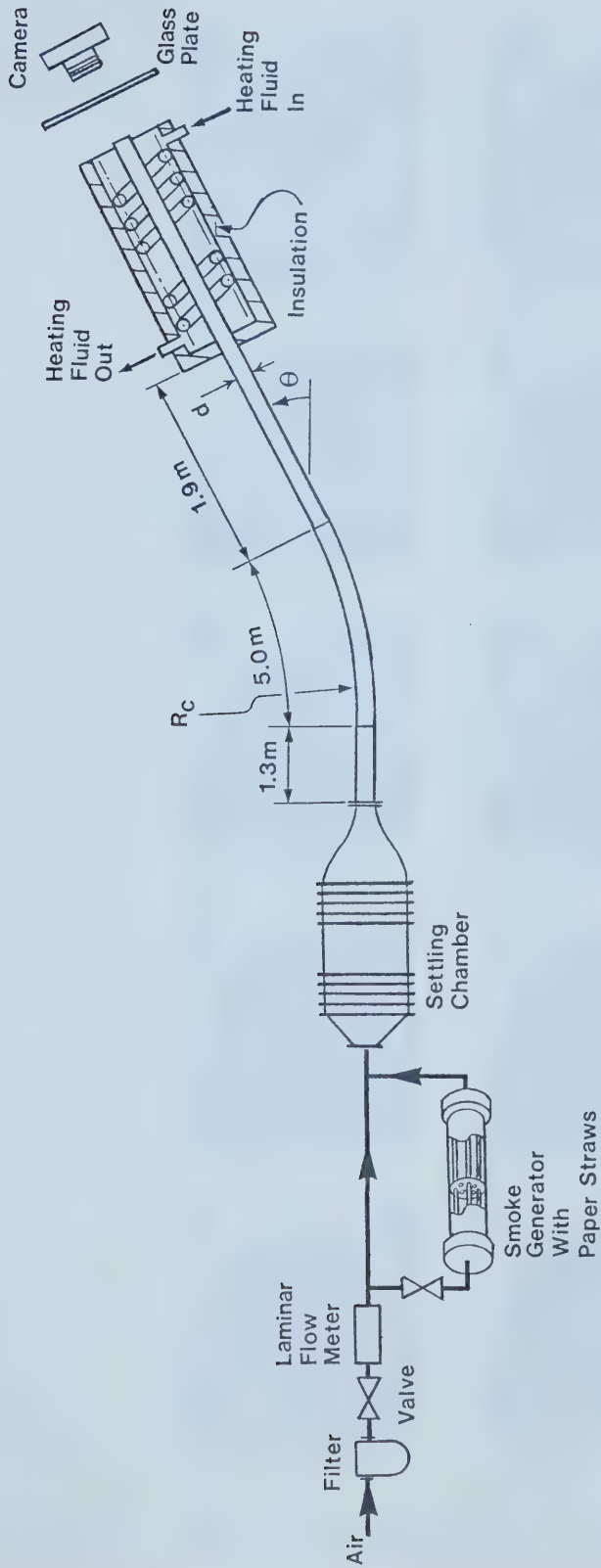


Fig. 6.1 Schematic diagram of experimental apparatus.

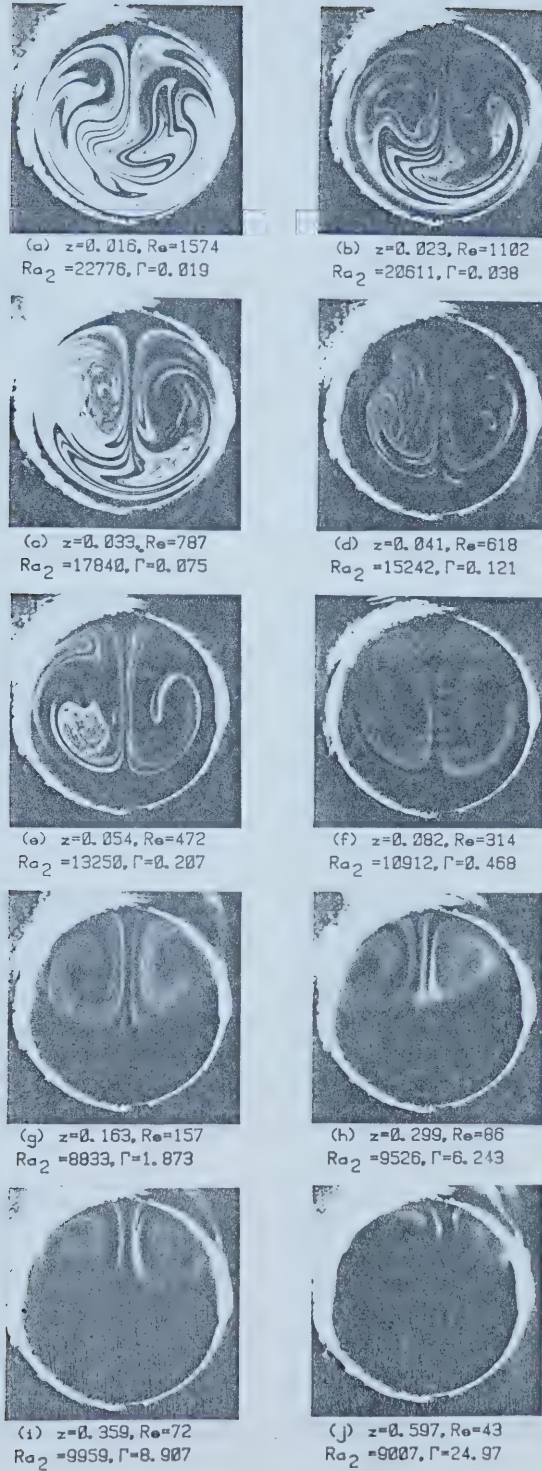


Fig. 6.2 Secondary flow patterns for $T_w=57^\circ\text{C}$, $Ra_1=3.27 \times 10^4$ and $\theta=30^\circ$.

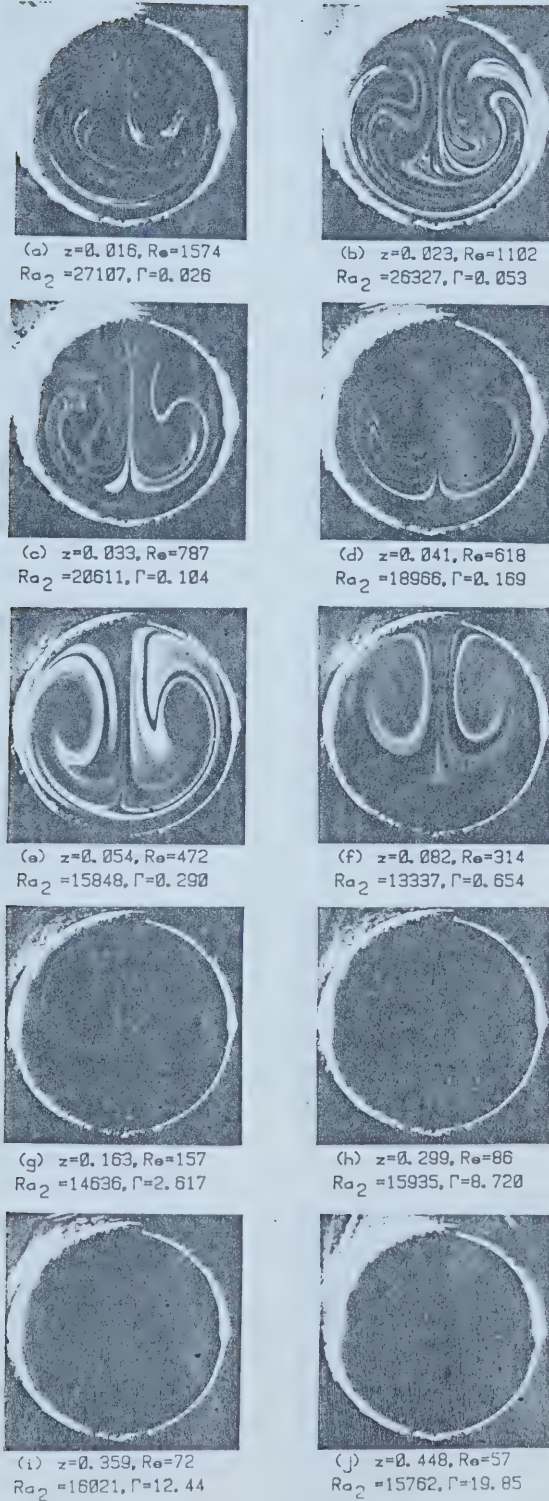


Fig. 6.3 Secondary flow patterns for $T_w=76^\circ\text{C}$, $Ra_1=4.57 \times 10^4$ and $\theta=30^\circ$.

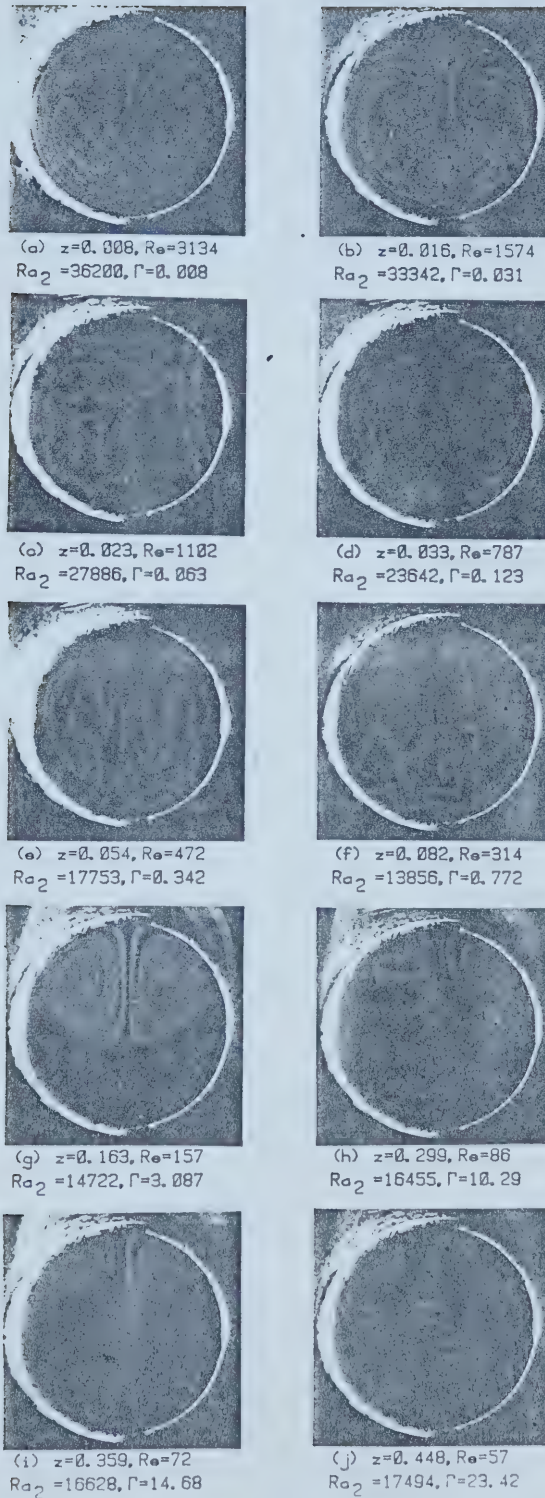


Fig. 6.4 Secondary flow patterns for $T_w=92^\circ\text{C}$, $Ra_1=5.40 \times 10^4$ and $\theta=30^\circ$.

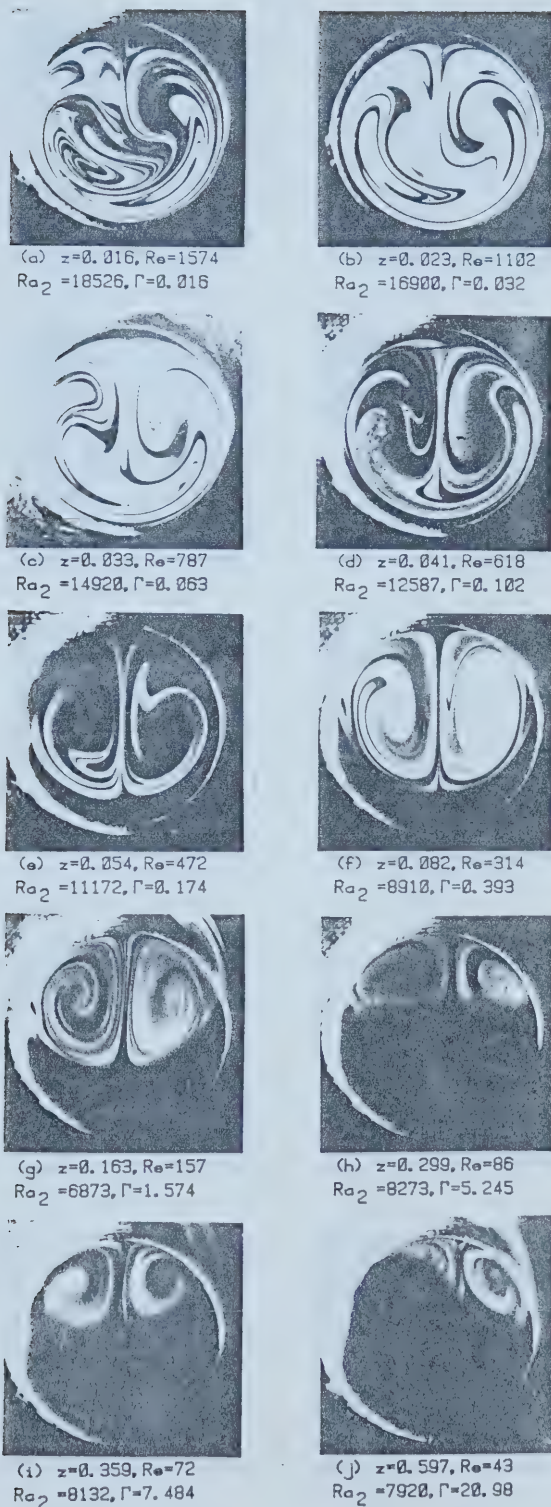


Fig. 6.5 Secondary flow patterns for $T_w=58^\circ\text{C}$, $Ra_1=2.75 \times 10^4$ and $\theta=45^\circ$.

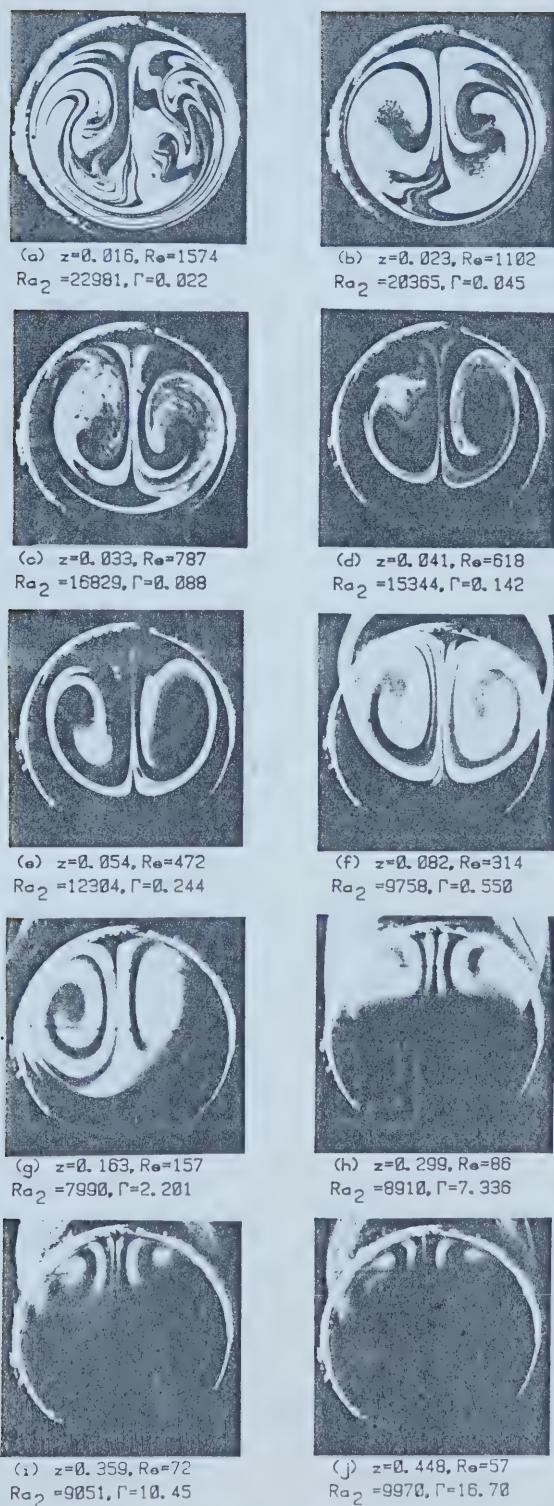


Fig. 6.6 Secondary flow patterns for $T_w=78^\circ\text{C}$, $Ra_1=3.85 \times 10^4$ and $\theta=45^\circ$.

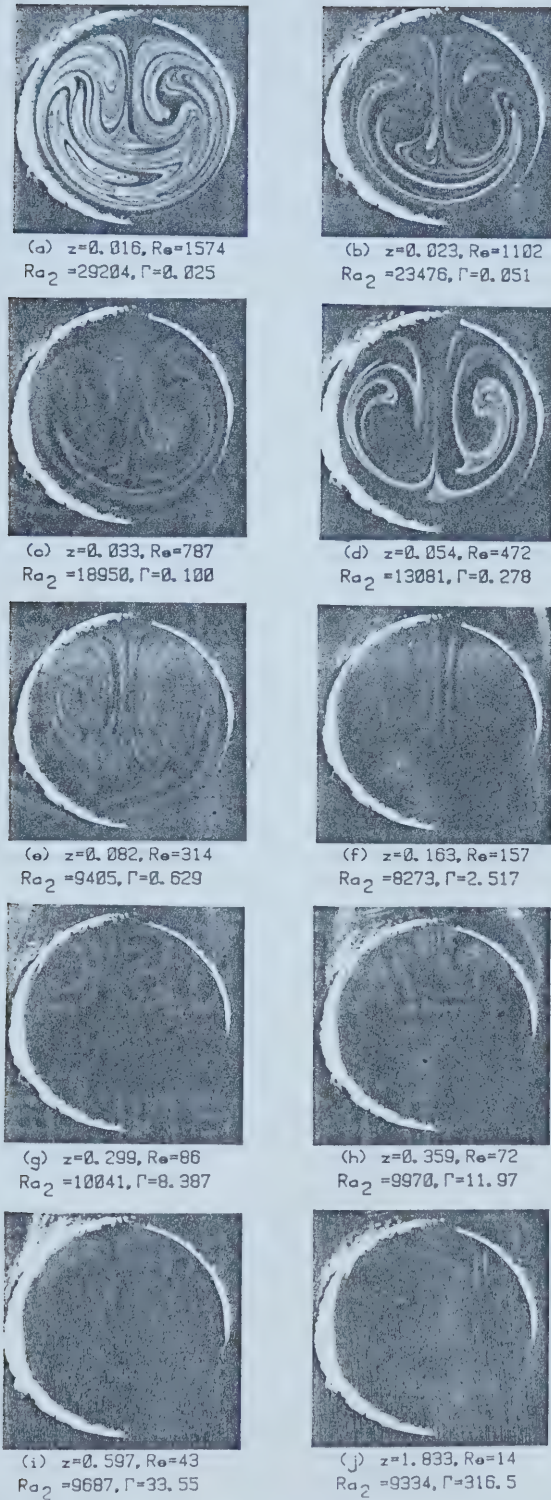


Fig. 6.7 Secondary flow patterns for $T_w=91^\circ\text{C}$, $Ra_1=4.40 \times 10^4$ and $\theta=45^\circ$.

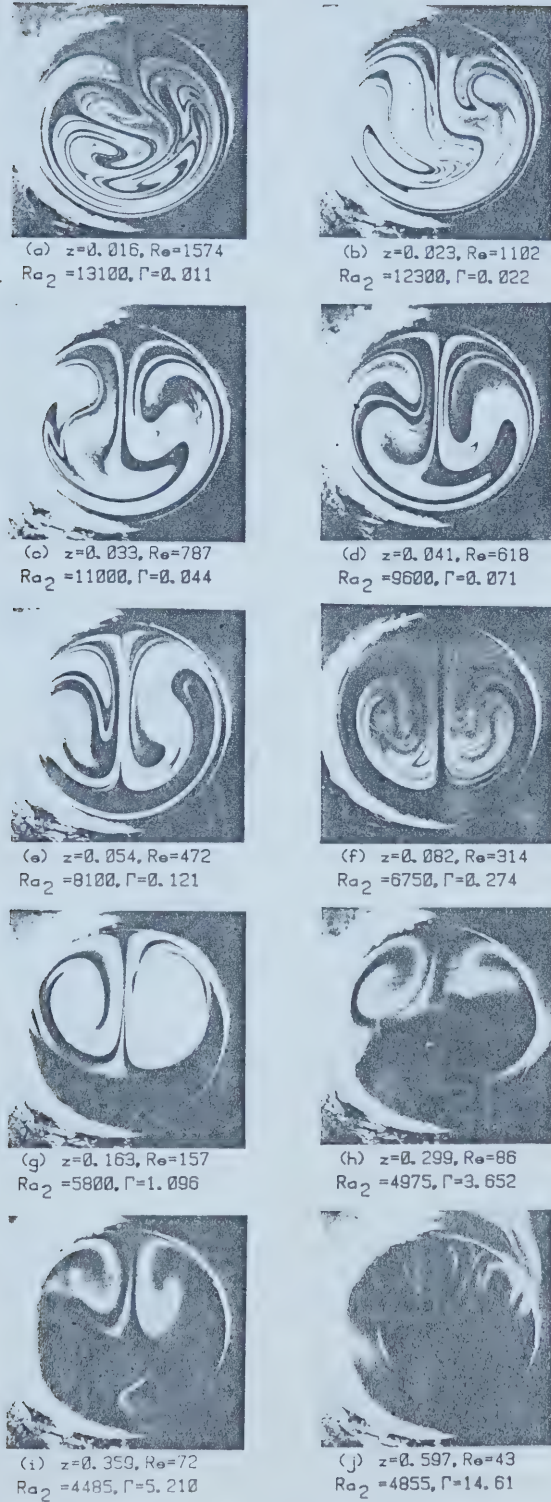


Fig. 6.8

Secondary flow patterns for $T_w=57^\circ\text{C}$, $Ra_1=1.92 \times 10^4$ and $\theta=60^\circ$.

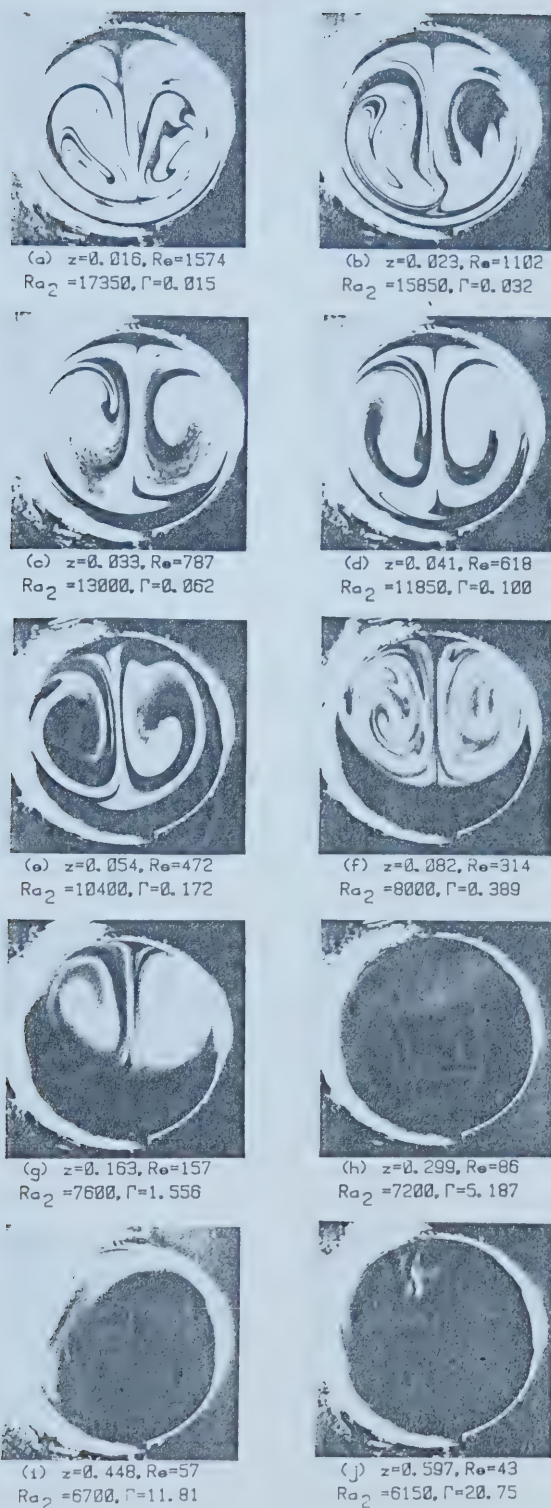


Fig. 6.9 Secondary flow patterns for $T_w=78^\circ\text{C}$, $Ra_1=2.72 \times 10^4$ and $\theta=60^\circ$.

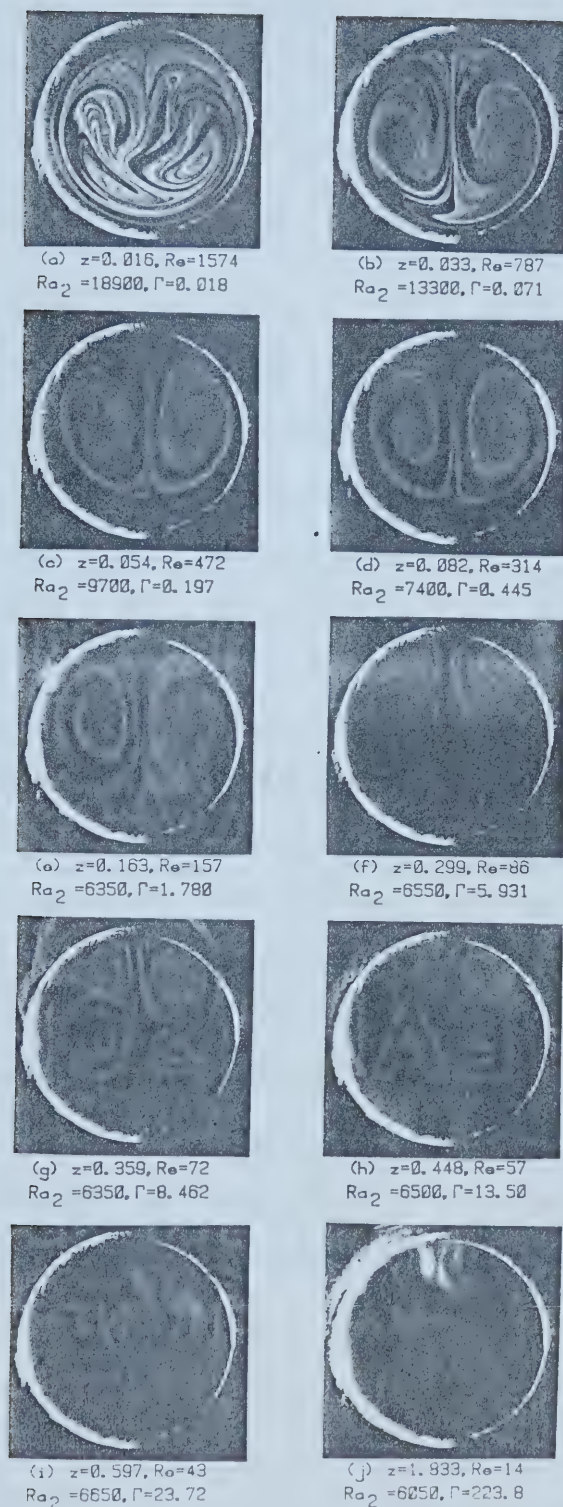


Fig. 6.10 Secondary flow patterns for $T_w=91^\circ\text{C}$, $Ra_1=3.11 \times 10^4$ and $\theta=60^\circ$.

7. Secondary Flow in an Isothermally Heated Curved Pipe⁶

Summary

Secondary flow patterns at the exit of a 180° bend (tube inside diameter $d=1.99\text{cm}$, radius of curvature $R_c=10.85\text{cm}$) are presented to illustrate the combined effects of centrifugal and buoyancy forces in hydrodynamically and thermally developing entrance region of an isothermally heated curved pipe with a parabolic entrance velocity profile. Three cases of upward, horizontal and downward curved pipe flows are studied for constant wall temperatures $T_w=55-91^\circ\text{C}$, Dean number range $K=22-1209$ and $ReRa=9.49 \times 10^5-8.86 \times 10^7$. Air at room temperature was used as the flowing fluid. The flow visualization was realized by the smoke injection method. The secondary flow patterns shown are useful for future comparison with numerical predictions and confirming theoretical models. The results can be used to assess the limit of the applicability of the existing correlation equations for laminar forced convection in isothermally heated curved pipes without buoyancy effect.

⁶A version of this chapter has been submitted for publication. Cheng, K.C. and Yuen, F.P. 106th ASME Winter Annual Meeting, Miami Beach, Florida, Oct., 1985.

7.1 Introduction

The thermal entrance region problem (Graetz problem) for fully developed laminar flow in a straight tube with constant wall temperature was solved first by Graetz[1] in 1885 and independently by Nusselt[2] in 1910. Historically, it is of particular interest to note that Graetz obtained the first analytical solution for developing thermal boundary layer inside a tube (parabolic problem) preceding the proposal of general laminar boundary layer concept for flow on a flat plate by Prandtl in 1904. The analytical solution for fully developed laminar flow in curved pipes was first obtained by Dean[3,4] in 1927 for very low Dean number flow regime. Fully developed free and forced laminar convection in uniformly heated horizontal pipes was solved by Morton [5] in 1959 for low Rayleigh number flow regime. In recent years, the buoyancy force effects on laminar forced convection in vertical and horizontal tubes and centrifugal force effects on laminar forced convection in curved pipes have been studied by many investigators both theoretically and experimentally. It is only recently that the combined buoyancy and centrifugal forces effects on laminar forced convection heat transfer have been reported in the literature [6-13]. The relevant references can be found in [6-13] and the literature review will not be repeated here.

The secondary flows caused by buoyancy forces in straight tubes and by centrifugal forces in curved pipes

have significant effects on laminar forced convective heat transfer. When the buoyancy effects are significant in curved pipes, one must consider the inclination of the plane of curvature and the direction of flow (upward or downward). Recent numerical and flow visualization studies [14-18] reveal the appearance of a four-vortex laminar flow in curved pipes at a certain Dean number. The additional pair of counter-rotating vortices is caused by centrifugal instability near concave outer wall and is known as Dean vortices. It is thus expected that the secondary flow patterns for laminar flow in heated or cooled curved pipes will be very complicated depending on the inclination of the plane of curvature and the direction of flow (upward or downward). It is expected that the secondary flow caused by both centrifugal and buoyancy forces will depend on Dean number K and the product of Reynolds and Rayleigh numbers, $ReRa$.

The purpose of this chapter is to investigate secondary flow patterns observed at the exit of a 180-degree bend (tube inside diameter $d=1.99\text{cm}$, radius of curvature $R_c=10.85\text{cm}$) for horizontal and vertical curved pipe configurations with parabolic laminar entrance flow and constant wall temperature $T_w=55-91^\circ\text{C}$. Air at room temperature was the flowing fluid. The flow visualization was made possible using smoke generated by burning a bundle of straw paper sticks in a smoke generator tube. Since flow visualization experiments were made by varying

Reynolds number with fixed curved pipe length (πR_c), the photographic results obtained represent the secondary flow patterns in the thermal entrance region of an isothermally heated curved pipe. The photographic results provide physical insight into the heat transfer mechanism for laminar force convection in curved pipes with buoyancy effects. Flow visualization studies are also useful in confirming the theoretical models for numerical and analytical solutions of mixed convection problems in curved pipes. The secondary flow patterns are presented to illustrate the simultaneous effects of centrifugal and buoyancy forces in the thermal entrance region of isothermally heated horizontal and vertical (upward or downward flow) curved pipes. It should be pointed out that theoretical (numerical and analytical) solution of the problem under consideration is extremely difficult.

7.2 Experimental Parameters

The experimental parameters for the present investigation are: curvature ratio $d/2R_c$, Reynolds number, Dean number, constant wall temperature, Rayleigh (or Grashof) number, inclination angle of the plane of curvature from the horizontal direction and direction of flow. For this investigation, only horizontal and vertical curved pipe configurations are considered. The entrance flow at the start of 180-degree bend was parabolic and in effect thermally and hydrodynamically developing flow in an

isothermally heated curved pipe was studied. The ranges of experimental parameters for this investigation are listed in Table 7.1.

Table 7.1 Ranges of Experimental Parameters

Air Flow Rate	18.9 - 1033.6 cm ³ /sec
Mean Velocity	6.05 - 331.0 cm/sec
Reynolds Number, Re	73 - 3989
ReRa	9.49×10^5 - 8.86×10^7
Rayleigh Number, Ra ₁	1.74×10^4 - 3.09×10^4
Dean Number, K	22 - 1209
Prandtl Number (Air)	0.71
Pipe Wall Temperature, Tw	55 - 91°C
$(d/2R_c)^{1/2}$	0.3032

7.3 Experimental Apparatus and Procedure

A schematic diagram of the experimental apparatus is shown in Fig. 7.1. Since the curved tube was rolled from a straight copper tube, the tube diameter at the exit of 180-degree bend was not uniform and the results of measurements are shown in Fig. 7.2 for reference. The water jacket for heating was made from a flexible heat-resistant

hose (o.d.=4.2cm). The hose was clamped on two 2.5cm long copper end pieces soldered on the tube bend forming the curved annular space for double-pipe heat exchanger. A helical coil was inserted in the annular space for heat transfer augmentation and the test section was insulated with 3.8cm Armaflex pipe insulation to prevent heat loss. Fig. 7.2 also shows an offset of 1.08cm between the end of water jacket and the centre of the radius of curvature. Hot water from a large constant temperature tank was circulated through the annular space in the parallel-flow direction to the air flow.

The inlet and outlet bulk temperatures of heating hot water were measured by 0.7mm o.d. sheathed iron-constantan thermocouples and the outlet centreline air temperature was measured by a type T thermocouple(o.d.=0.254mm). All thermocouples were initially calibrated using a quartz thermometer (HP-2807A) with a maximum error of $\pm 0.3^{\circ}\text{C}$. Since the temperature difference between the inlet and outlet bulk temperatures of the hot water were found to be rather small considering the measurement error, the average value was taken as the average tube wall temperature, T_w .

As shown in Fig. 7.1, a straight entrance length of 3.2m was provided between the settling chamber and the start of 180° bend to ensure a fully developed parabolic velocity profile at the entrance of the test section. Room air from compressor was used and the air flow rate was measured by a Meriam laminar flow element which employs a Dwyer inclined

manometer. The experimental error was estimated to be $\pm 1.75\%$. Flow visualization was made possible by burning a bundle of paper straws inside a smoke generator tube located in front of the settling chamber.

The experimental technique was similar to that used in earlier investigations [17-19]. By passing a sheet of light provided by a 500W slide projector, the secondary flow pattern can be observed at the exit of the isothermally heated curved tube section. The heated air was discharged directly into the atmosphere as a jet. Three different cases of horizontal curved pipe, vertical curved pipe with upward flow and vertical curved pipe with downward flow as shown in Fig. 7.2 were studied. A Nikon FM2 single lens reflex camera with a 50 mm micro lens was used together with Kodak Tri-X black and white film and camera settings of f3.5 and $1/2 - 1/8$ sec.

7.4 Results and Discussion

The main interest here is to study the secondary flow patterns resulting from the interaction between centrifugal and buoyancy forces for three different geometrical configurations (vertical curved pipe with upward flow, horizontal curved pipe flow and vertical curved pipe with downward flow). Physically, the Dean number K can be considered as the ratio of the centrifugal force to the viscous force and the parameter $ReRa$ represents the ratio of the buoyancy force to the viscous force.

In contrast to the theoretical results [7,8] for similar problems reported in the literature, the flow visualization technique using the smoke injection method enables one to study a wider range of parametric values and the ranges $22 \leq K \leq 1209$ and $9.49 \times 10^5 \leq ReRa \leq 8.86 \times 10^7$ were reached in this investigation.

Since the heated curved pipe length is fixed, an increase in the parameter $ReRa$ for a constant wall temperature may be regarded as an increase in Reynolds number or a decrease in the dimensionless axial distance from the thermal entrance, $(l/d)/RePr$, which is an inverse Graetz number representing the thermal entrance length. It is noted that no provision was made to prevent axial heat conduction through the copper tube from the heated section in the upstream direction. The photographic results for secondary flow patterns for laminar flows in the unheated curved pipe and isothermally heated horizontal pipe are presented in [18].

7.4.1 Photographs for Vertical Curved Pipe with Upward Flow, Figs. 7.3 to 7.5

The photographs for secondary flow pattern are arranged in the order of decreasing Dean numbers in Figs. 7.3, 7.4 and 7.5 for the average wall temperatures of 57, 76 and 91°C, respectively. The directions of the centrifugal forces and buoyancy forces are coplanar but they may not be in phase depending on the angular position ϕ from the start

of bend. Without buoyancy force effect, the secondary flow develops gradually from the start of bending to the end of 180° bend. It is generally understood that the centrifugal forces induce a pair of symmetric counter-rotating vortices in a cross-section normal to the main flow and the fluid in the central core moves towards the concave outer wall. The buoyancy force effect may be negligible at the start of bending (see Fig. 7.2). Its effect increases with the angle of bend but decreases to zero momentarily at 90° bend. The effect then increases again from the 90° bend position to the 180° bend position. The buoyancy forces also induce a pair of symmetric vortices.

The secondary flows caused by centrifugal and buoyancy forces are in phase in the region $\phi=0$ and 90° but are 180° out of phase in the region $\phi=90$ and 180° . The intensity of secondary flow due to the centrifugal forces increases monotonically in the 180° bend until a fully developed flow is reached. The discussion here is only qualitative since the interaction between the two regions of vortices may occur due to internal or external disturbances. The secondary flow pattern observed at the exit of 180° bend represents the effects of the complicated interaction process of centrifugal and buoyancy forces in the 180° bend section.

In Figs. 7.3 to 7.5, the top of each photograph represents the outside of bend and the bottom represents the inside of bend. In photographs (a) and (b) of Fig. 7.3, one

sees the dividing streamline in the lower part caused by centrifugal forces. In the upper part the centrifugal forces and buoyancy forces are apparently of the same order of magnitude resulting in a rather complicated secondary flow with several vortices seen clearly and no symmetry can be observed. At $K=850$ in (a), the Reynolds number is 2805 exceeding the critical Reynolds number of 2300 for straight tube flow. It is seen that a laminar flow still persists at $K=850$. The secondary flow patterns shown in (c), (d), (e) and (f) can be identified as somewhat similar to those of isothermal flow in curved pipes [17,18] and one may infer that the centrifugal forces are dominating over the buoyancy forces for the parametric values shown.

At $K \leq 149$ in Figs. 7.3 to 7.5, apparently the buoyancy forces dominate over the centrifugal forces in (g), (h), (i) and (j) and the symmetric secondary flow patterns are similar to those observed in thermal entrance region of an isothermally heated horizontal tube [18]. The black crescent region signifies the vanishing of the secondary flow due to buoyancy forces. At $K=22$, the centrifugal force effect is very weak and the curvature effect may be negligible practically. When the secondary flow disappears completely, a fully developed flow for the limiting Nusselt number $Nu_{\infty}=3.66$ is approached for the constant wall temperature case.

At $K=850$ in Figs. 7.4 and 7.5, the buoyancy forces are seen to be dominating over the centrifugal forces and the

secondary flow patterns are not very clear probably due to smoke diffusion at a high secondary flow velocity. The effects of Dean number and the values of $ReRa$ on secondary flow patterns for wall temperatures $T_w=76$ and 91°C are qualitatively similar to those at $T_w=57^\circ\text{C}$. It is of interest to note that the secondary flow pattern at $K=850$ represents that near the thermal entrance with small inverse Graetz number and $K=22$ represents secondary flow pattern at larger inverse Graetz number. At $K=425$ and 607 in Figs. 7.3 to 7.5, one sees clearly the eyes of an additional pair of vortices located near the outer wall of the bend and the interaction between the left-hand and right-hand regions is appreciable in the upper part of the curved pipe. At $K \leq 182$, the line of symmetry is quite straight and the buoyancy forces are dominant over the centrifugal forces.

7.4.2 Photographs for Horizontal Curved Pipe Flow, Figs. 7.6 to 7.8

This case is of considerable practical interest since many cooling or heating coils are used in this geometrical configuration. Intuitively, this case can be considered as a combination of centrifugal force effect in an isothermal flow in a curved tube and buoyancy effect in an isothermally heated horizontal straight tube. The secondary flow patterns for the average wall temperatures $T_w=56$, 76 and 91°C are shown in Figs. 7.6 to 7.8, respectively. In each photograph, the left-hand side represents the concave outer

wall of the bend and the right-hand side represents the convex inner wall of the bend. For the horizontal curved pipe shown (see Fig. 7.2), the buoyancy forces act in the upward direction and the centrifugal forces act towards the left and the two forces are always perpendicular to each other in the 180° bend section. The secondary flow patterns at the exit of the 180° bend represent the outcome of a hydrodynamically and thermally developing entrance region.

The secondary flow patterns in Fig. 7.6 for $T_w=56^\circ\text{C}$ will be examined first. At $K=850$ and 729 , the centrifugal force effect can be seen from the dividing streamline near the right-hand wall (inner bend). The flow pattern near the outer wall is not very clear at $K=850$ due to smoke diffusion but several vortices are clearly seen at $K=729$. At $K=607$, one observes six vortices and four vortices remain at $K=425$. One also sees a heated air layer exists near the outer wall and the trend becomes more apparent at $K=304$ and 243 where only one cell remains near the outer wall. The flow pattern is more complicated in the region near the outer wall where the centrifugal instability occurs. The additional vortices or cell near the outer wall in Figs. 7.6(b) to (f) are caused by centrifugal instability but are subject to buoyancy effect.

It appears that the centrifugal forces are dominant over the buoyancy forces for $K=850$, 729 , 607 and 425 . As Dean number K decreases from $K=304$ to 22 , the buoyancy effect increases gradually over the centrifugal force effect

and the distortion of the dividing streamline due to the centrifugal forces is of particular interest. At $K=61$, a crescent region (black part) appears near the lower wall and the area of the region with secondary flow decreases with further decrease of Dean number. The lifting of the secondary flow pattern is due to buoyancy force effect.

The centrifugal and buoyancy force effects on secondary flow patterns at wall temperatures $T_w=76$ and 91°C are qualitatively similar to the case with $T_w=56^\circ\text{C}$ and the effect of wall temperature increase can be seen in Figs. 7.7 and 7.8. In studying the secondary flow pattern, it is of interest to note the distortion of the dividing streamline caused by buoyancy effect. For $K=607$ to 182 in Fig. 7.7, the dividing streamline is diverted upward near the outer wall and is lifted near the inner wall. At $K=243$, the secondary flow patterns are quite similar for $T_w=56$, 76 and 91°C . The appearance of the crescent region and the lifting of the secondary flow region with decreasing area is the characteristic feature of the buoyancy effect for small Dean number flow regime. The secondary flow in isothermally heated pipes show no symmetry since two body forces act in perpendicular directions. At $K=1033$ in Fig. 7.7, the Reynolds number is $Re=3406$ exceeding the critical Reynolds number $Re=2300$ but the flow is still laminar in the presence of both centrifugal and buoyancy force effects.

7.4.3 Photographs for Vertical Curved Pipe with Downward Flow, Figs. 7.9 to 7.12

Intuitively, this case can be considered as a combination of centrifugal force effect in an isothermal flow in a curved tube and buoyancy effect in an isothermally heated vertical tube with downward flow. The secondary flow patterns for the wall temperatures $T_w=55, 76, 85$ and 91°C are presented in Figs. 7.9 to 7.12, respectively. In each photograph, the top represents the convex inner wall and the bottom represents the concave outer wall of the bend. At any angular position in the initial 90° bend ($\phi=0-90^\circ$), the centrifugal forces and buoyancy forces are acting in opposite direction but they are acting in the same direction in the bend section between $\phi=90^\circ$ and 180° . The buoyancy effect is momentarily zero at $\phi=90^\circ$.

In Fig. 7.9 with $T_w=55^\circ\text{C}$, the secondary flow patterns in the Dean number range $K=729$ to 61 clearly show the centrifugal force effect as evidenced by the dividing streamline and the vortex pattern. At $K=966$, the buoyancy and centrifugal effects are apparently of the same order of magnitude and the secondary flow pattern is rather complicated. Generally the flow pattern is more complicated in the lower part where the centrifugal instability exists near the outer wall. At $K=61$, a crescent region (black part) already appears near the lower wall and the secondary flow region decreases with further decrease of Dean number. The lifting of the secondary region is due to buoyancy

effect.

In Fig. 7.10 with $T_w=76^\circ\text{C}$, the centrifugal force effects are clearly seen in the Dean number range $K=607$ to 243 and a layer of heated air near the lower wall due to buoyancy forces is also seen. At $K=850$, the dividing streamline cannot penetrate into the lower part. A crescent region already appears near the lower wall at $K=61$.

In Fig. 7.11 with $T_w=85^\circ\text{C}$, the centrifugal force effects are clearly seen in the secondary flow patterns for $K=607$ to 121 . The distortion of the secondary flow pattern from that of isothermal flow in a curved pipe suggests that at $K=966$ and 850 , the buoyancy effect is appreciable. At $K=149$ and 121 , one sees the four-vortex pattern and a pair of counter-rotating vortices near the lower (outer) wall can be identified as Dean vortices caused by centrifugal instability in the region near the concave wall. Since Dean vortices are not observed for the cases $T_w=55$ and 76°C (see Figs. 7.9 and 7.10) in the same Dean number range, one may conclude that the Dean vortices are triggered by buoyancy forces at higher wall temperature. Since no external disturbances are required for the onset of four-vortex secondary flow pattern, it is concluded that the presence of the buoyancy forces in this case could be a source of disturbances. At $K=110$, the buoyancy and centrifugal forces may be of the same order of magnitude and the secondary flow pattern is similar to the familiar secondary flow pattern caused by a single body force (centrifugal or buoyancy

force) in lower parameter flow regime. At $K=61$, a crescent region appears near the lower wall. At $K=22$, the secondary flow is confined to a small region near the upper wall and the fully developed region with $Nu_{\infty}=3.66$ for constant wall temperature is approached.

In Fig. 7.12 with $T_w=91^{\circ}\text{C}$, the buoyancy effects are seen to be appreciable in the Dean number range $K=966-425$ and the secondary flow patterns are complex. The heated air layers near the side walls are of interest. In the Dean number range $K=215$ to 149 , the four-vortex secondary flow patterns are seen with a good symmetry. At $K=121$, two heated air layers can be seen extending from the bottom to the top. At $K=61$, a crescent region appears near the bottom and the lifting effect of the buoyancy forces can be seen.

Although not depicted in Fig. 7.10, the unsteady, oscillating Dean vortices were observed in the range $110 \leq K \leq 180$ for $T_w=76^{\circ}\text{C}$. At higher wall temperature, the Dean vortices become more stable over a wider Dean number range. The onset of the four-vortex secondary flow pattern is of considerable theoretical interest. The experimental data based on flow visualization are shown in Fig. 7.13 for reference. The experimental data are too few to define an instability region for the onset of four-vortex flow pattern. It is of interest to note that the four-vortex secondary flow pattern was not observed in a vertical curved pipe with upward flow.

7.5 Concluding Remarks

The secondary flow patterns at the exit of a 180° bend are presented to illustrate the combined effects of centrifugal and buoyancy forces in hydrodynamically and thermally developing entrance region of an isothermally heated curved pipe with parabolic entrance velocity profile for the following three cases: (1) Vertical curved pipe with upward flow, (2) Horizontal curved pipe flow, and (3) Vertical curved pipe with downward flow.

The Dean vortices are observed only in the downward flow inside a curved vertical pipe and the onset of Dean vortices occurs at around $K=121$ and $ReRa \geq 6.66 \times 10^6$. The case of convection heat transfer in a horizontal curved pipe is of considerable practical interest. The secondary flow patterns for the case of horizontal curved pipe are skewed since the buoyancy and centrifugal forces are perpendicular to each other.

The present flow visualization results provide physical insight into the convective heat transfer mechanism inside an isothermally heated curved pipe with buoyancy and centrifugal force effects. The present photographic results are believed to be valuable for future theoretical and experimental studies on heated and cooled curved pipes.

7.6 References

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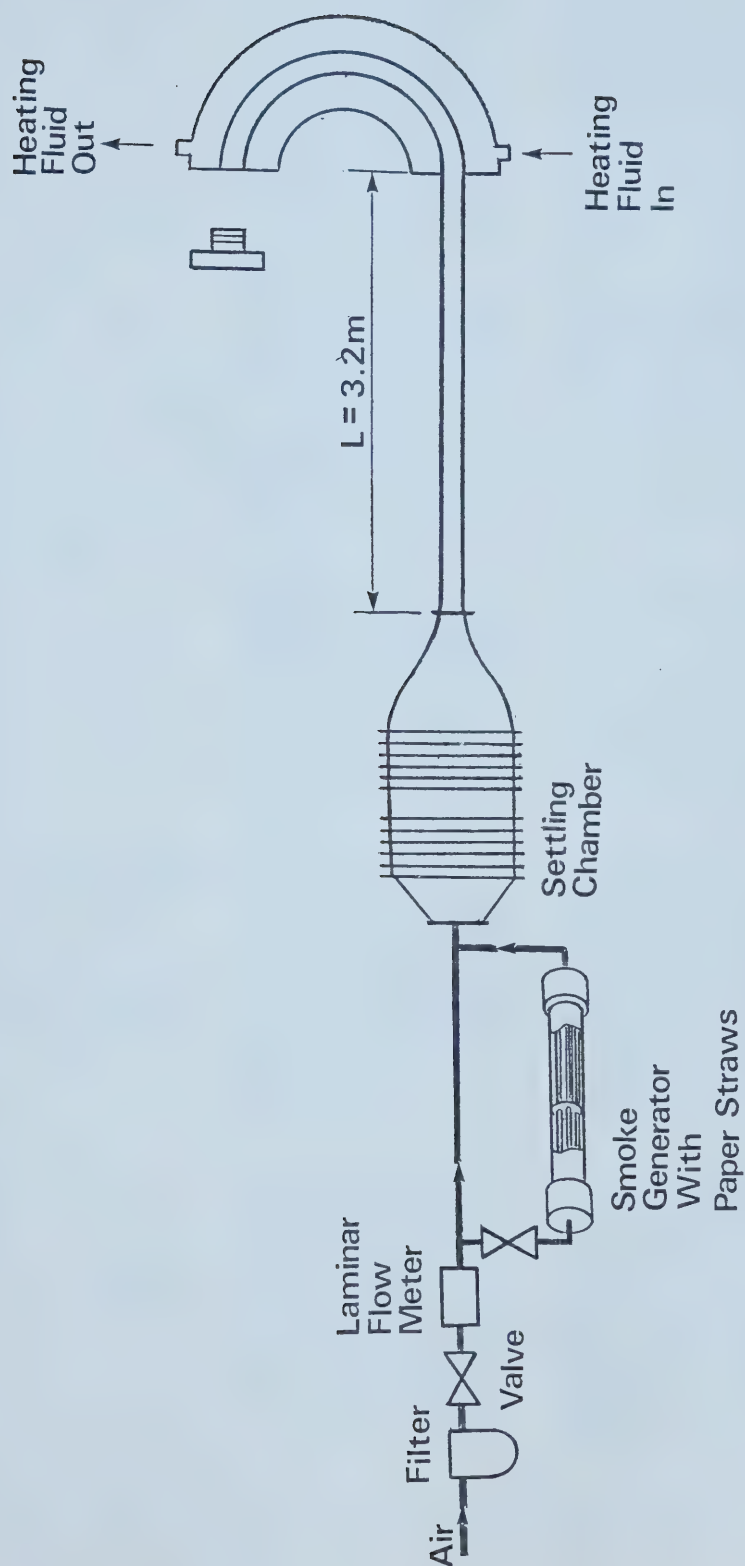


Fig. 7.1 Schematic diagram of experimental apparatus.

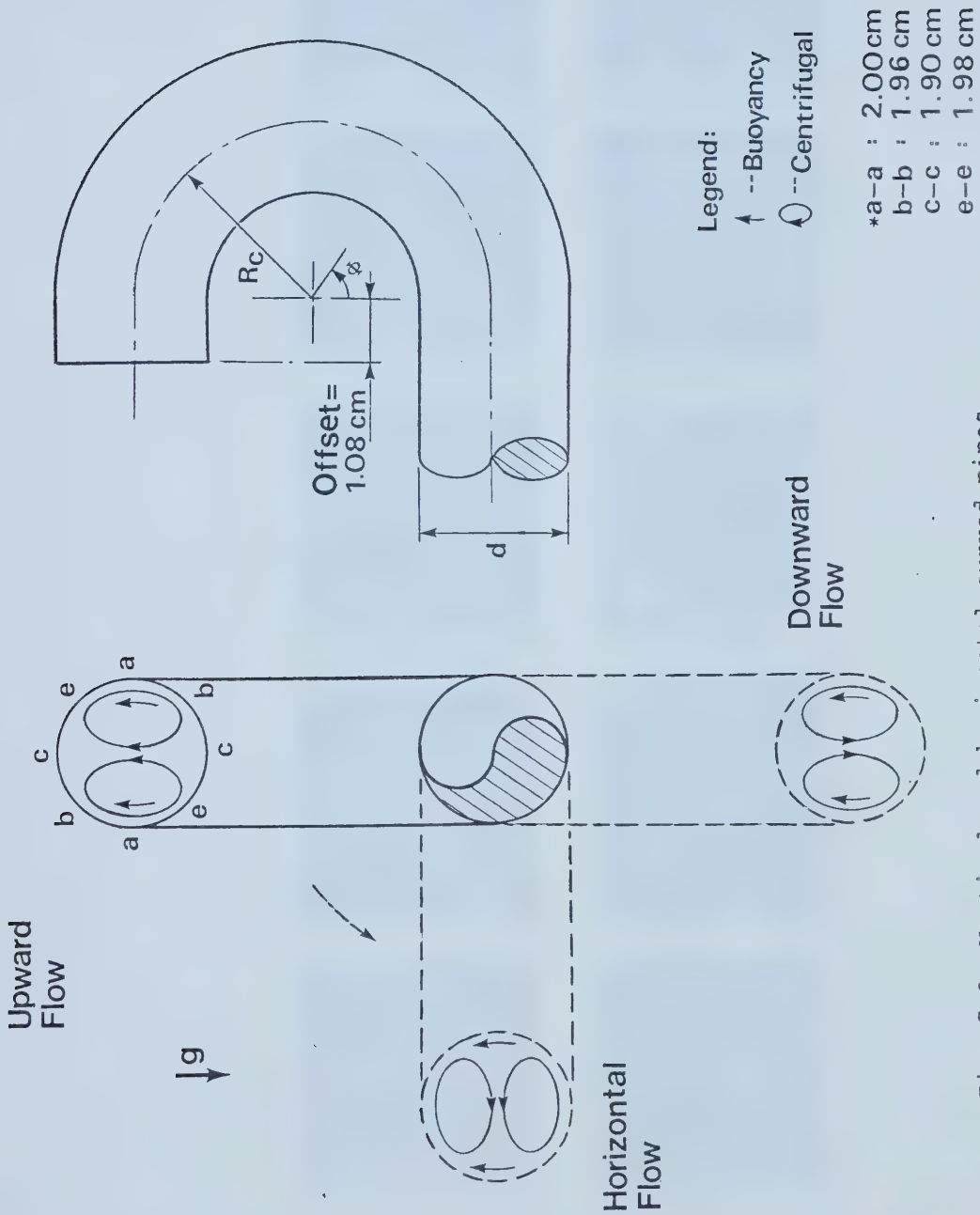


Fig. 7.2 Vertical and horizontal curved pipes.

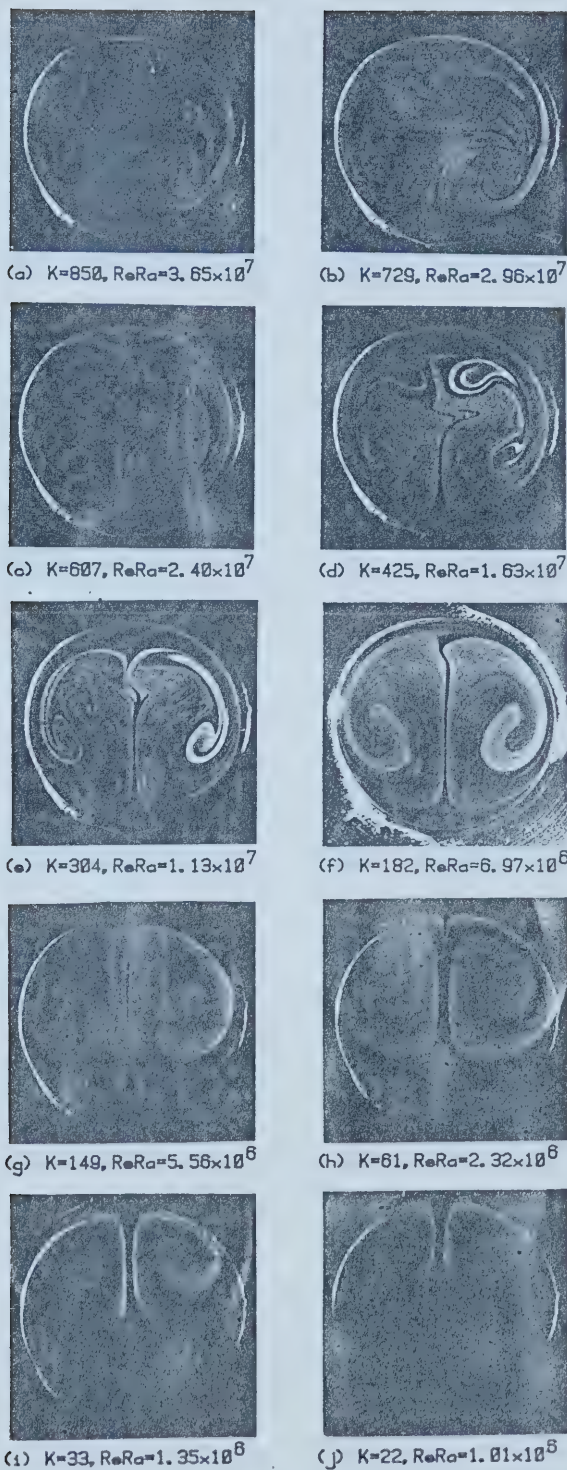


Fig. 7.3 Secondary flow patterns in a vertical curved pipe with upward flow for $T_w=57^\circ\text{C}$ and $Ra_1=1.85 \times 10^4$.



Fig. 7.4 Secondary flow patterns in a vertical curved pipe with upward flow for $T_w=76^\circ\text{C}$ and $Ra_1=2.57 \times 10^4$.

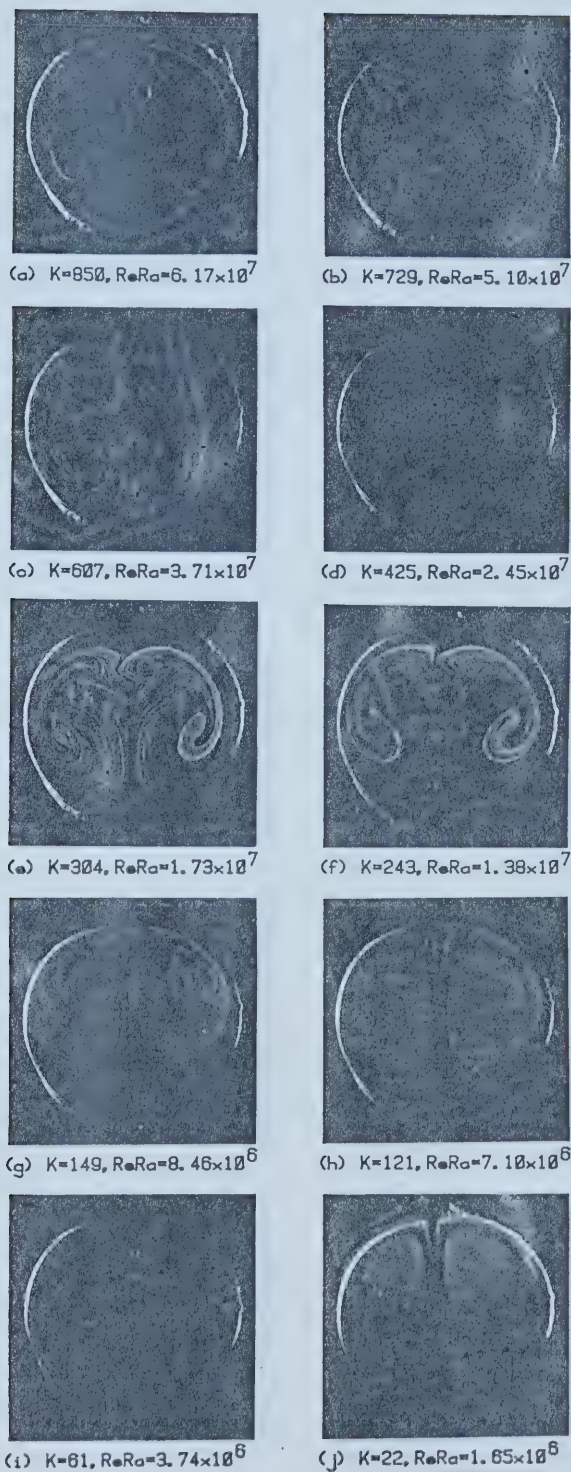


Fig. 7.5 Secondary flow patterns in a vertical curved pipe with upward flow for $T_w=91^\circ\text{C}$ and $Ra_1=3.02 \times 10^4$.



Fig. 7.6 Secondary flow patterns in a horizontal curved pipe for $T_w=56^\circ\text{C}$ and $Ra_1=1.80 \times 10^4$.

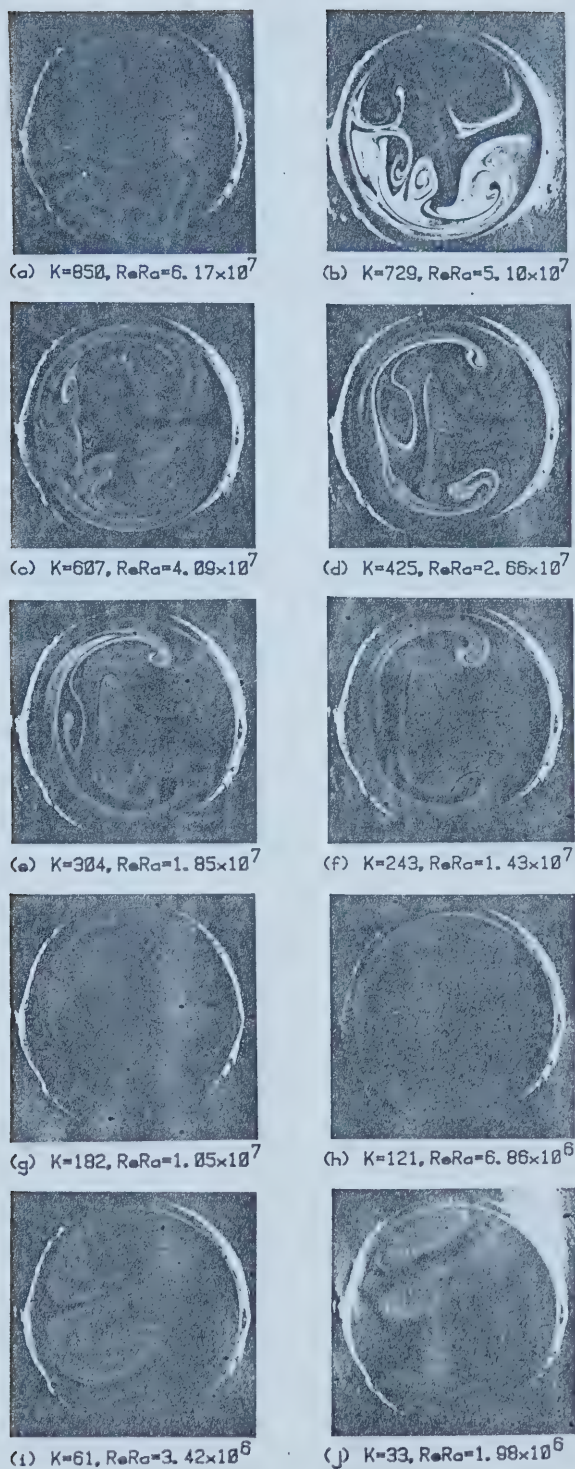


Fig. 7.7 Secondary flow patterns in a horizontal curved pipe for $T_w=76^\circ\text{C}$ and $Ra_1=2.57 \times 10^4$.

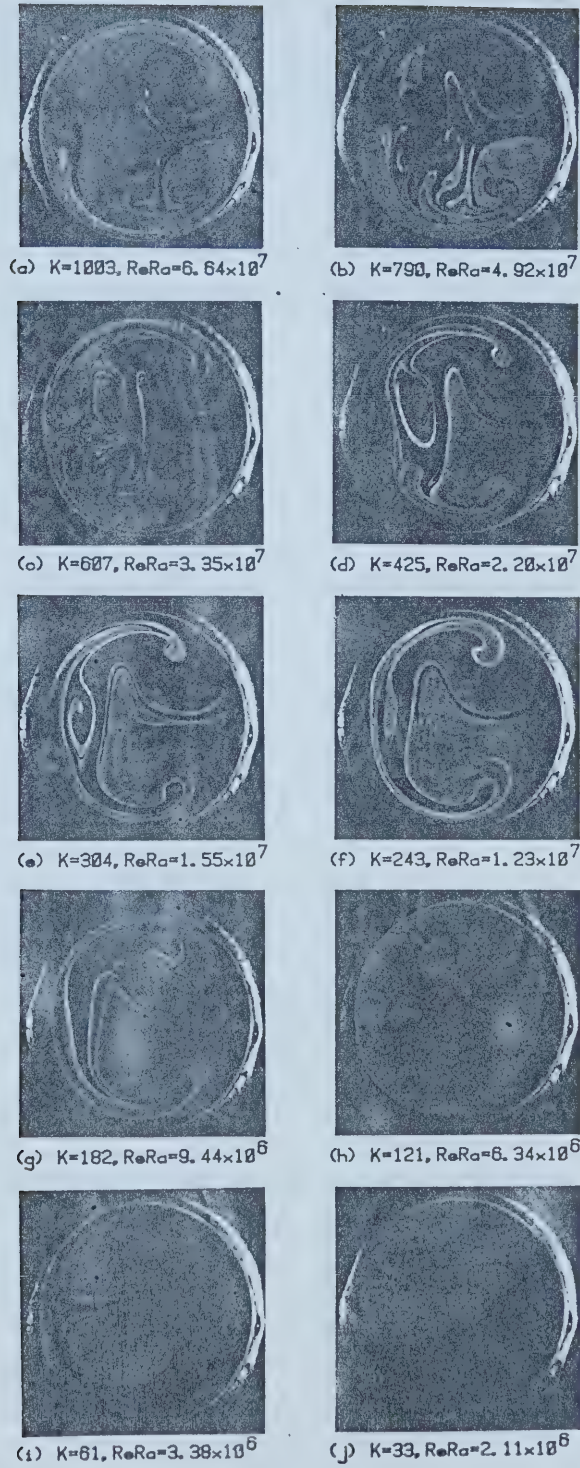


Fig. 7.8 Secondary flow patterns in a horizontal curved pipe for $T_w=91^\circ\text{C}$ and $Ra_1=3.02 \times 10^4$.

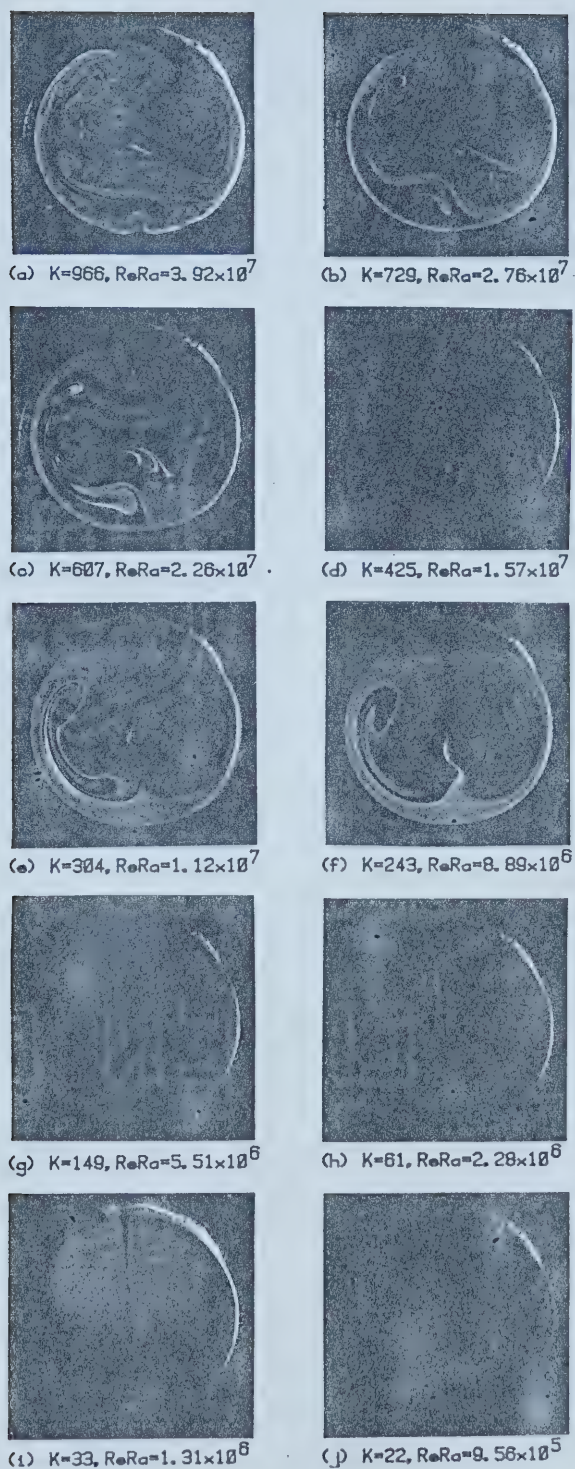


Fig. 7.9 Secondary flow patterns in a vertical curved pipe with downward flow for $T_w=55^\circ\text{C}$ and $Ra_1=1.78 \times 10^4$.

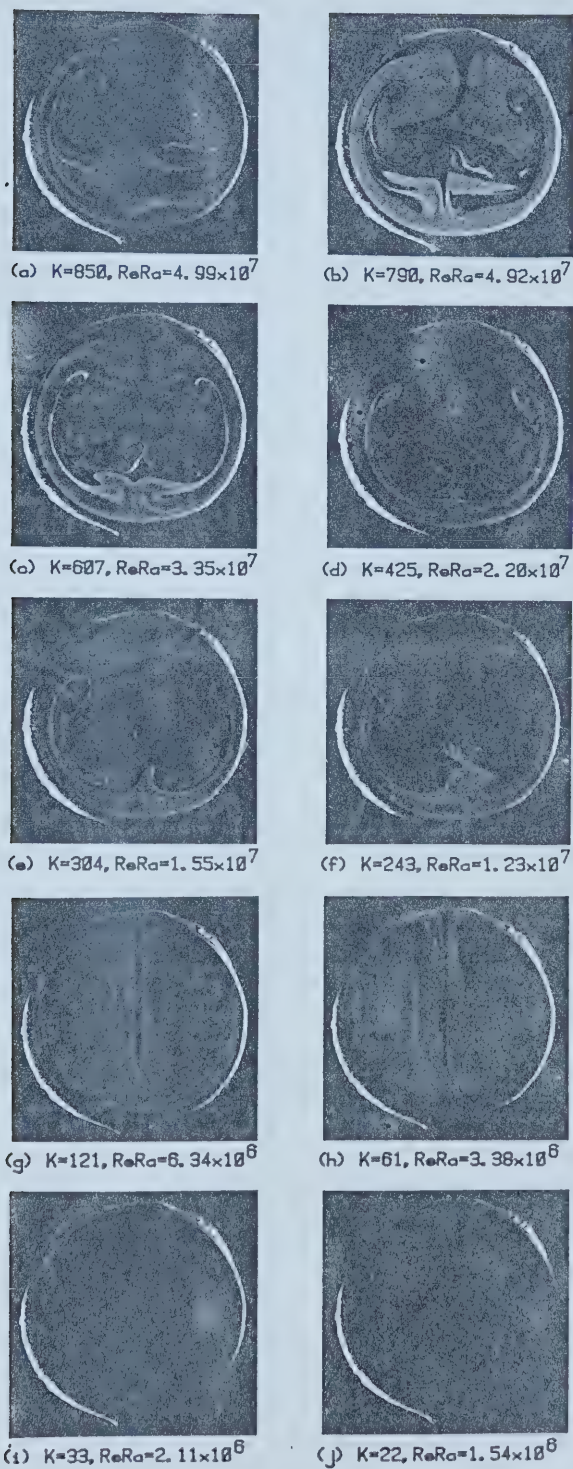


Fig. 7.10 Secondary flow patterns in a vertical curved pipe with downward flow for $T_w=76^\circ\text{C}$ and $Ra_1=2.57 \times 10^4$.

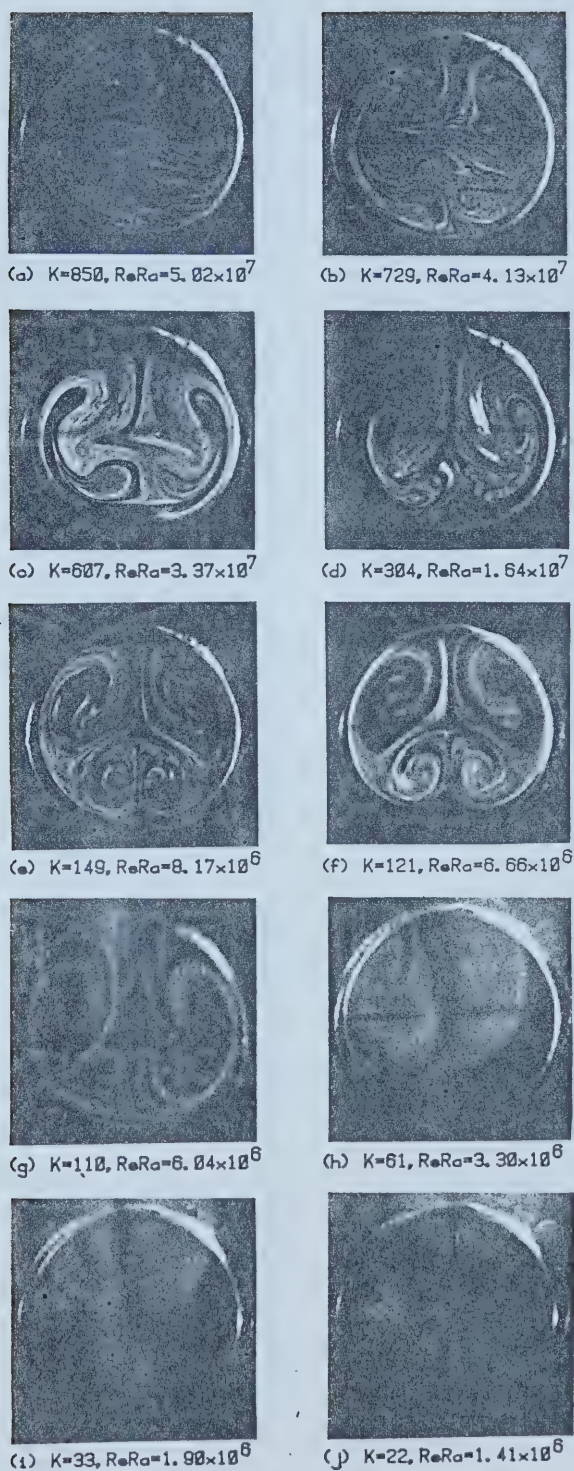


Fig. 7.11 Secondary flow patterns in a vertical curved pipe with downward flow for $T_w=85^\circ\text{C}$ and $Ra_1=2.84 \times 10^4$.

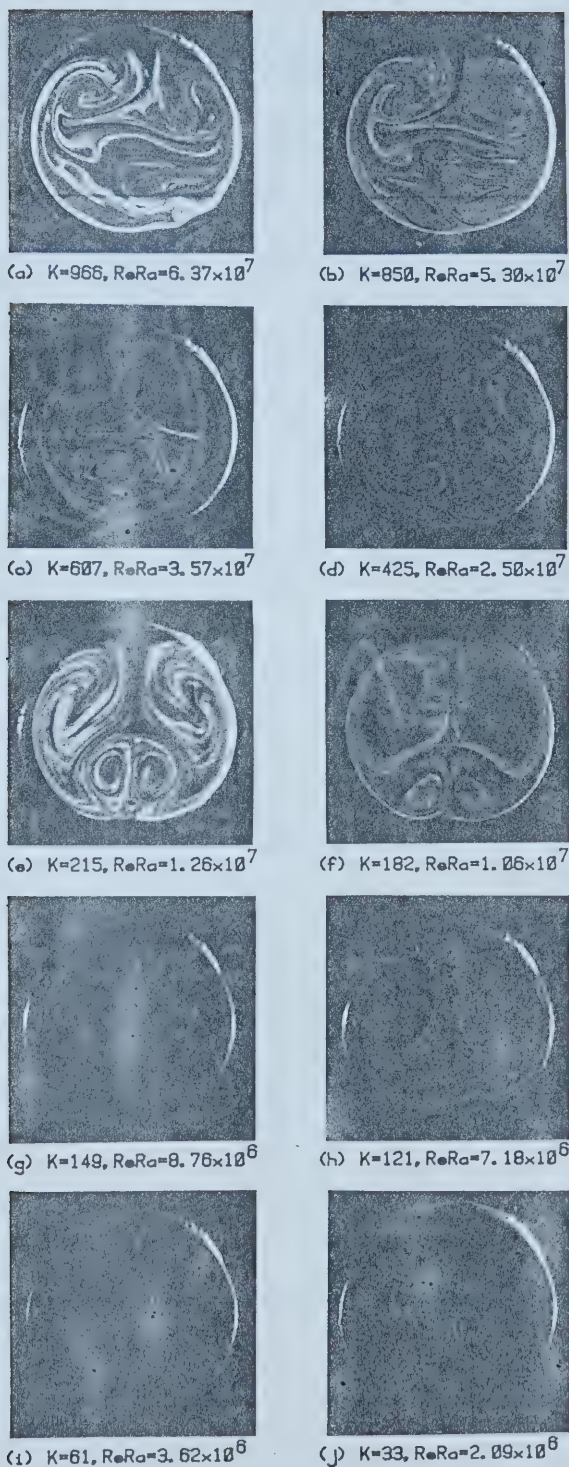


Fig. 7.12 Secondary flow patterns in a vertical curved pipe with downward flow for $T_w=91^\circ\text{C}$ and $Ra_1=3.02 \times 10^4$.

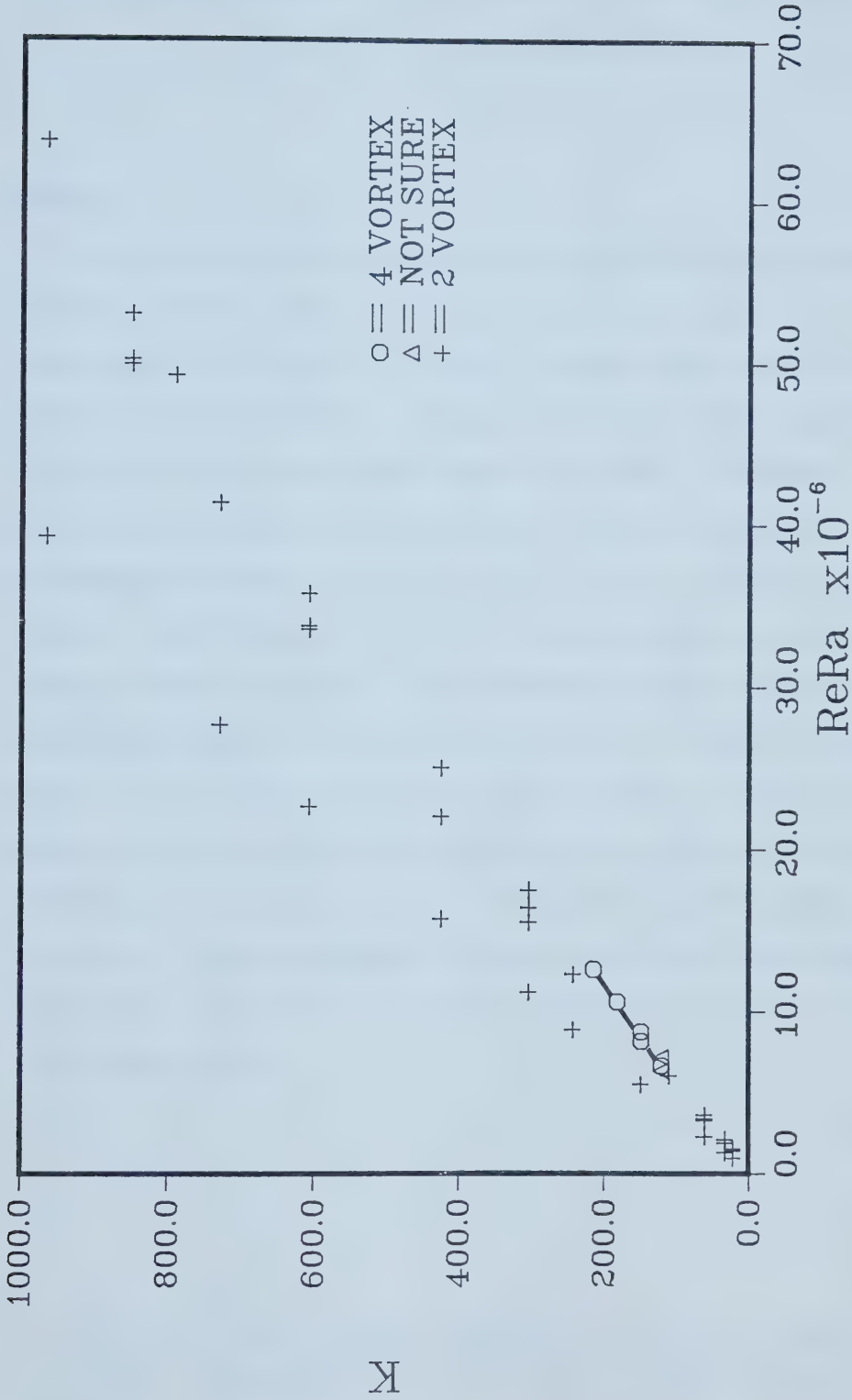


Fig. 7.13 The onset of four-vortex secondary flow pattern.

8. Secondary Flow at the Outlet and in the Downstream Region of a Centrifugal Blower⁷

Summary

Photographs are presented for secondary flow patterns and axial flow patterns in a circular duct section downstream ($d=3.17\text{cm}$) of a small centrifugal fan with a fan speed range $n=0-640$ rpm. Experiments have been carried out in: a straight tube downstream ($d=3.17\text{cm}$), a sudden enlargement ($d_2=5.11\text{cm}$), and two conical diffusers with divergence angles $\beta_1=6.0^\circ$ and 14.1° after the circular fan outlet. Flow visualization was made possible using the smoke injection method. The secondary flow patterns are also presented for the free outlets of two small centrifugal fans with circular and rectangular outlets. The varying complicated secondary flow patterns and flow separation phenomena in the conical diffusers are of particular interest. The photographic results provide some insight into three-dimensional unsteady turbulent flow inside centrifugal fans.

⁷A version of this chapter has been published. Cheng, K.C. and Yuen, F.P.. Preprints, Symposium on Transport Phenomena in Rotating Machinery, Honolulu, Hawaii, Apr. 28-May 3, 1985.

8.1 Introduction

Blowers (centrifugal fans) and fans have a wide range of applications. They can be used for exhausting or introducing air or gas; assisting combustion in furnaces; conveying pneumatically; or simply ventilating for comfort and safety. They are also regarded as the heart of air distribution systems. The fluid motion inside the blower is characterized by three-dimensional, turbulent and extremely unsteady flow. Because of the effects of centrifugal and Coriolis forces generated by the scroll-type or volute-type fan housing and rotating blades inside a centrifugal fan, one expects a strong secondary flow at the outlet and in the downstream region of the blower. Since theoretical analysis of the internal fluid motion from the inlet to the exit of a centrifugal fan is extremely difficult, it is felt that flow visualization studies on the secondary flow patterns after the discharge are of considerable practical interest.

The purpose of this chapter is to observe secondary flow patterns at the free outlet and in the downstream region of a small centrifugal fan at the fan speed range $n=0-640$ rpm. Three cases of outlet ducts are studied: a straight duct, a sudden enlargement ($d_2=5.11$ in) and two conical diffusers. The secondary flow patterns at the free outlet of a small centrifugal fan with a rectangular outlet are also presented.

Flow visualization of internal flow in rotating machinery has been studied by many investigators and review

articles are available [1-4]. Recently, flow visualizations of a sirocco fan [5] and a cross fan [6-8] were also reported. Although our flow visualization technique using the smoke injection method is limited to a relatively low exit velocity, the secondary flow patterns presented provide some insight into three-dimensional unsteady fluid motion after the discharge of a centrifugal fan. The decay of secondary flow in a straight downstream pipe and conical diffusers with divergence angles $\beta_1=6.0^\circ$ and 14.1° are also studied.

8.2 Experimental Apparatus and Procedure

The experimental setup for flow visualization is quite straight forward and a schematic diagram is shown in Figure 8.1. Two small shaded pole centrifugal blowers, shown in Figure 8.2(a), were chosen for the studies and their specifications are given in Table 8.1.

The downstream ducts were made of plexiglas to make visualization possible. The two different sizes of conical diffusers with lengths $l=24.21\text{cm}$ and 7.58cm , and $\beta_1=6.0^\circ$ and 14.1° respectively were also machined from plexiglas. Both fans are equipped with speed controllers. Centrifugal fan speeds were measured by a variable flash rate stroboscope and the local centreline velocity (V_c) was measured by a TSI air velocity meter (Model 1650).

The flow visualization was facilitated by burning a bundle of Chinese incense sticks near the free inlet of the

Table 8.1 Specifications of the Centrifugal Blowers

Make and Model	Dayton 2C782	Dayton 4C723
Power Rating	1/250 hp	1/45 hp
Free Air RPM	3160	2300
Outlet Size	3.17 cm dia.	7 x 4.5 cm
Inlet Size	4.76 cm dia.	11.9 cm dia.
Casing	Scroll-Type	Volute-Type
CFM Free Air	15(0.42 m ³ /min.)	82(2.32 m ³ /min.)
Cut-Off SP	0.22" (0.56cm)	1.35" (3.43cm)
No. of Blades	24	26
Blade Types	Forward-curved	Forward-curved

centrifugal fan with forward-curved blades. Air enters the inlet axially and is discharged tangentially. A slit light source was provided by a 500W slide projector. The secondary flow patterns were observed at the outlets ($x=0$) of the centrifugal fans and at various downstream sections of the circular fan. The plan (top) and side views of the axial flow in the downstream section were also photographed. Over 1200 photographs were taken using a Nikon FM2 SLR camera with a 55mm micro lens and Kodak Tri-X Pan film using a f3.5 aperture and 1/2-1/15 sec. shutter speed settings.

8.3 Experimental Parameters

In this investigation, the flow rate of the centrifugal fan was not measured. The centreline velocity at the outlet of the fan or downstream tube was measured for reference only. Because of the effect of smoke diffusion at high exit velocity, the smoke injection method for flow visualization was restricted to relatively a low fan rpm range. For the centrifugal fan with a circular outlet, the measured maximum rpm was 3000 and the centreline exit velocity was $V_c=4.8\text{m/s}$. The experimental ranges for flow visualization were $n=0-640\text{rpm}$ and $V_c=0-0.77\text{m/s}$. The experimental ranges were restricted to $n=0-305\text{rpm}$ and $V_c=0-1.7\text{m/s}$. Flow visualization in the downstream region was performed only for the centrifugal fan with a circular outlet. Two conical diffusers with $d_1=3.17\text{cm}$, $d_2=5.04\text{cm}$, $l=7.58\text{cm}$, $\beta_1=14.1^\circ$, $l/d=2.52$ and $d_1=3.20\text{cm}$, $d_2=5.75\text{cm}$, $l=24.21\text{cm}$, $\beta_1=6.0^\circ$, $l/d=7.57$ were used. For the downstream tube with a sudden enlargement at the fan outlet: $d_1=3.17\text{cm}$, $d_2=5.11\text{cm}$ and $d_2/d_1=1.61$. For the centrifugal fan with a circular outlet, the Reynolds number (Re) based on $V_c=0.77\text{m/s}$ and outlet diameter d_1 was $Re=1480$ and good photographs were obtained up to $Re=826$ ($V_c=0.43\text{m/s}$). For the centrifugal fan with the rectangular outlet, the Reynolds number based on $V_c=1.7\text{m/s}$ and hydraulic diameter $De=5.48\text{cm}$ was $Re=5628$ and good photographs were obtained up to $Re=1655$ ($V_c=0.5\text{m/s}$).

8.4 Results and Discussion

8.4.1 Photographs at the Exit with Discharge into Atmosphere

The velocity profile at the outlet of a centrifugal fan is not uniform. Typical secondary flow patterns at the exit of a centrifugal fan are shown in Figs. 8.3 and 8.4 for circular and rectangular outlets, respectively. At a fan speed of $\text{rpm}=0$, one can see a natural flow of smoke through the fan. A pair of Dean-type vortices is clearly seen near the top in Fig. 8.3 (a) in addition to a pair of large vortices. The secondary flow pattern changes at different rpm's and several different types of vortices can be observed. At $n=640\text{rpm}$, the smoke injection method does not yield a clear secondary flow pattern due to smoke diffusion at higher exit velocities. For the rectangular outlet, a pair of vortices appears near the top at $n=0$ due to a natural flow of smoke through the fan. At $n=85\text{rpm}$, two corner vortices are seen near the lower wall and persist up to $n=155\text{rpm}$. At $n=220\text{rpm}$, the corner vortices cannot be detected.

8.4.2 Photographs in the Ducted Outlet with Constant Inside Diameter ($d=3.17\text{cm}$)

The secondary flow patterns in the outlet tube with constant diameter ($d=3.17\text{cm}$) are shown in Fig. 8.5 at $n=0-390\text{rpm}$ and the downstream distance x from the fan outlet for $x=0-10d$. At $x=0$, one observes four or more vortices

depending on the fan speed. At $n=0$, the secondary flow pattern is fairly symmetric. For $n>0$, symmetry cannot be maintained. This may be due to the fact that a mounting screw (about 1 cm long) was protruding at the inside casing wall near the outlet. The impeller offset in the casing may also have been another contributing factor.

The secondary flow is unsteady and the photographs shown represent essentially the instantaneous patterns. Although the results are not shown here, the secondary flow is confirmed up to $n=640\text{rpm}$. Due to the upward moving secondary flow in the core region, maximum axial velocity is located near the upper wall. Since the Reynolds number is less than 2300, the secondary flow and turbulence generated by the impeller are expected to decay after a sufficiently long outlet tube. The present results seem to suggest that the secondary flow will persist for some distance before reaching a fully developed parabolic profile for a low rpm range. At $x=10d$, the secondary flow appears to be more symmetric in all cases.

8.4.3 Photographs in the Downstream Region after Sudden Enlargement

The secondary flow patterns in the downstream region after a sudden enlargement of a diameter from $d=3.17\text{cm}$ to $d_2=5.11\text{cm}$ (diameter ratio=1.61) at the fan outlet are shown in Fig. 8.6 with a fan speed of $n=250\text{rpm}$. For $x\leq 3d$ ($d=3.17\text{cm}$), the effect of a sudden increase in diameter

appears to be appreciably large and many vortices can be seen. At $x=10d$, the region is occupied only by a pair of vortices. It is interesting to note a gap near the lower wall at $x=5d$ and $7d$ which may be an indication of the flow separation.

The behavior of axial flow in the downstream region is also presented. By illuminating the central vertical or horizontal plane with a sheet of light, a side or plan view of the main flow in the downstream region can be obtained. These results are shown in Figs 8.7 and 8.8 for $n=0-390\text{rpm}$ and $n=0-250\text{rpm}$ respectively. Fig. 8.7 reveals that at $n=140$ and 250rpm , a wave-like flow exists in the central region. Furthermore, the flow pattern at $n=250\text{ rpm}$ seems to confirm the flow separation at $x=5d$ and $7d$ in Fig. 8.6. Figs. 8.7 and 8.8 also confirm an appreciable change in the flow pattern after the sudden enlargement of the cross-sectional area. Fig. 8.2(b) shows the top view of the recirculating zone near the shoulder.

8.4.4 Photographs in Two Conical Diffusers

The secondary flow patterns in the conical diffuser with $l=24.21\text{cm}$ and $\beta_1=6.0^\circ$ are shown in Fig. 8.9 for $n=140$ and 250rpm . The side and plan views of the axial flow in the conical section are shown in Figs. 8.10 and 8.11 respectively. Fig. 8.9 clearly reveals the flow separation phenomenon in the lower region and can be confirmed from the axial flow pattern in Fig. 8.10. At $n=140\text{rpm}$, a

laminar-like flow can be seen but at $n=250\text{rpm}$, the secondary flow is more complicated and appreciable turbulence can be seen. One notes that the plan view in Fig. 8.11 reveals no separation phenomenon in the upper half region of the conical diffuser.

The secondary flow patterns observed at the exit of the two conical diffusers are shown in Fig. 8.12. Photographs on the left-hand side are for $l=7.58\text{cm}$ and $\beta_1=14.1^\circ$; and the right-hand side for $l=24.21\text{cm}$ and $\beta_1=6.0^\circ$. The side and plan views of the axial flow in the conical diffuser with $l=7.58\text{cm}$, $\beta_1=14.1^\circ$ are shown in Fig. 8.13. It is helpful to compare the photographs on the left-hand side of Fig. 8.12 with the side views of Fig. 8.13 to understand the flow separation phenomenon. The secondary flow patterns shown in Fig. 8.12 are characterized by rather complicated multiple vortices of different sizes and flow separation near the lower wall. Fig. 8.2(c) shows the top view of axial flow in conical diffuser ($\beta_1=14.1^\circ$) with a pair of decaying vortices leaving the diffuser.

8.5 Concluding Remarks

The secondary flow at the outlet of a centrifugal fan represents an integrated result of complicated flow from the inlet of the fan where theoretical analysis would be extremely difficult. A simple flow visualization experiment using the smoke injection method readily yields the secondary flow pattern downstream at the outlet of a small

centrifugal fan. Fan speed ranges from 0-640rpm.

The secondary flow patterns in the downstream region are observed with the following types of outlet duct: a straight tube, a sudden enlargement and two conical diffusers with divergence angles $\beta_1=6.0^\circ$ and 14.1° . The flow patterns for the axial flow along the vertical and horizontal central planes are also obtained for the outlet sections. The flow separation phenomena in the conical diffusers are clearly observed.

The presented flow visualization study for very low fan speeds shows that the development of the entry flow from the fan outlet to a fully developed laminar flow with parabolic profile is extremely complicated problem and cannot be readily approached theoretically.

This flow visualization study does not yield any quantitative results but confirms the existence of unsteady secondary flow. The limit of the applicability of the smoke injection method is shown and other flow visualization methods should be used for future work with $V_c > 0.6 \text{ m/s}$.

8.6 References

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8. Takahashi, K. and Diakoku, T., "Flow Visualization of Cross-Flow Fan", J. of the Flow Visualization Soc. of Japan, Vol. 2, No. 6, 1982, pp. 473-478.

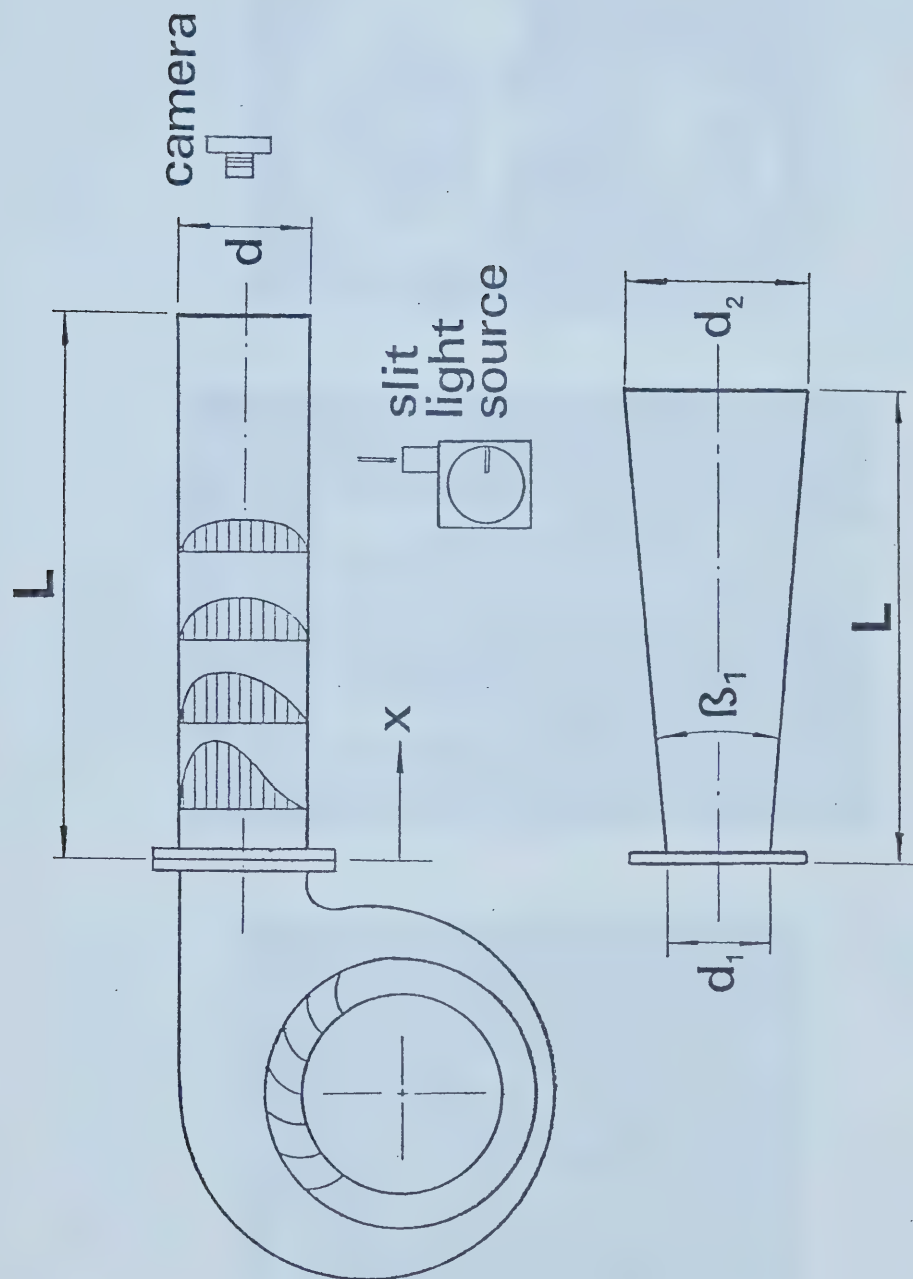


Fig. 8.1 Schematic diagram of experimental apparatus.

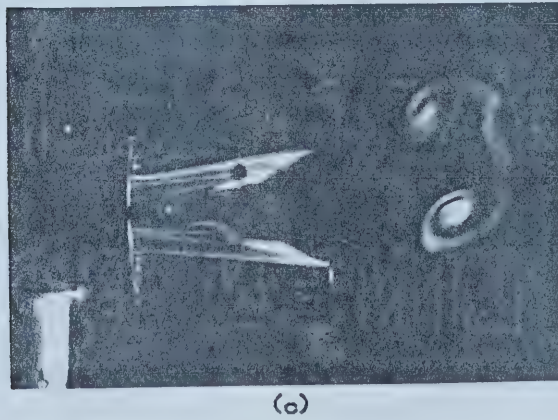
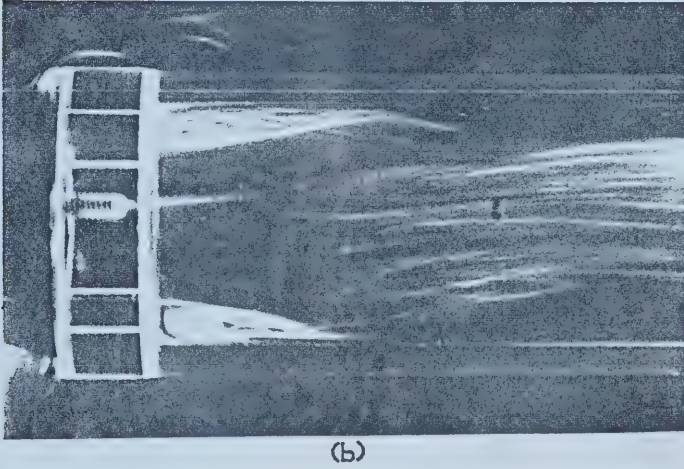
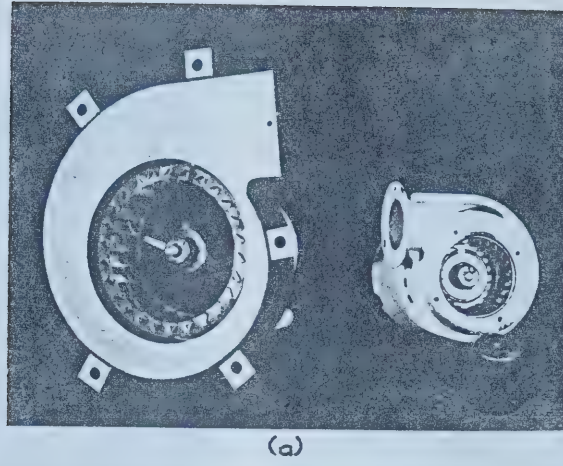


Fig. 8.2 Close-up views of two tested fans and their axial flow patterns.



(a) $n=0$ RPM, $V_c=0$ m/s



(b) $n=170$ RPM, $V_c=0.24$ m/s



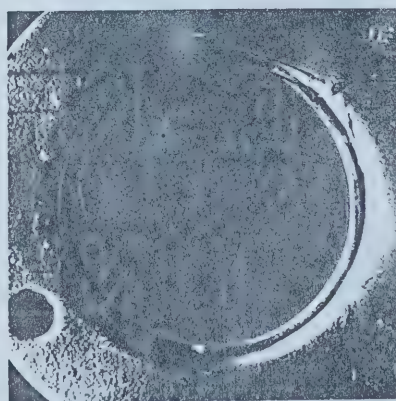
(c) $n=250$ RPM, $V_c=0.31$ m/s



(d) $n=340$ RPM, $V_c=0.43$ m/s



(e) $n=390$ RPM, $V_c=0.55$ m/s



(f) $n=640$ RPM, $V_c=0.77$ m/s

Fig. 8.3 Secondary flow patterns at the circular outlet.

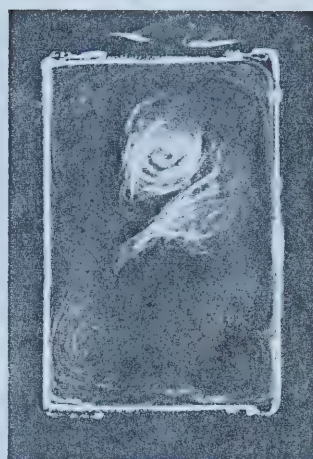
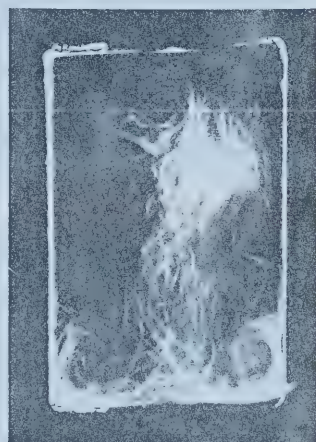
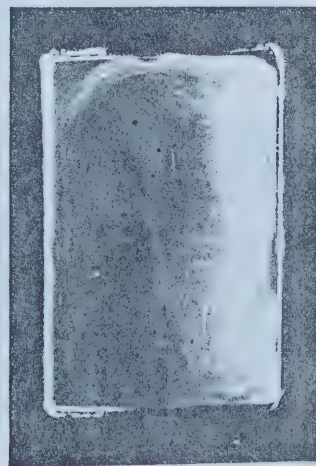
(a) $n=0\text{rpm}$, $V_c=0$ (b) $n=85\text{rpm}$, $V_c=0.35\text{m/s}$ (c) $n=110\text{rpm}$, $V_c=0.50\text{m/s}$ (d) $n=130\text{rpm}$, $V_c=0.60\text{m/s}$ (e) $n=155\text{rpm}$, $V_c=0.75\text{m/s}$ (f) $n=220\text{rpm}$, $V_c=1.10\text{m/s}$

Fig. 8.4 Secondary flow patterns at the rectangular outlet.

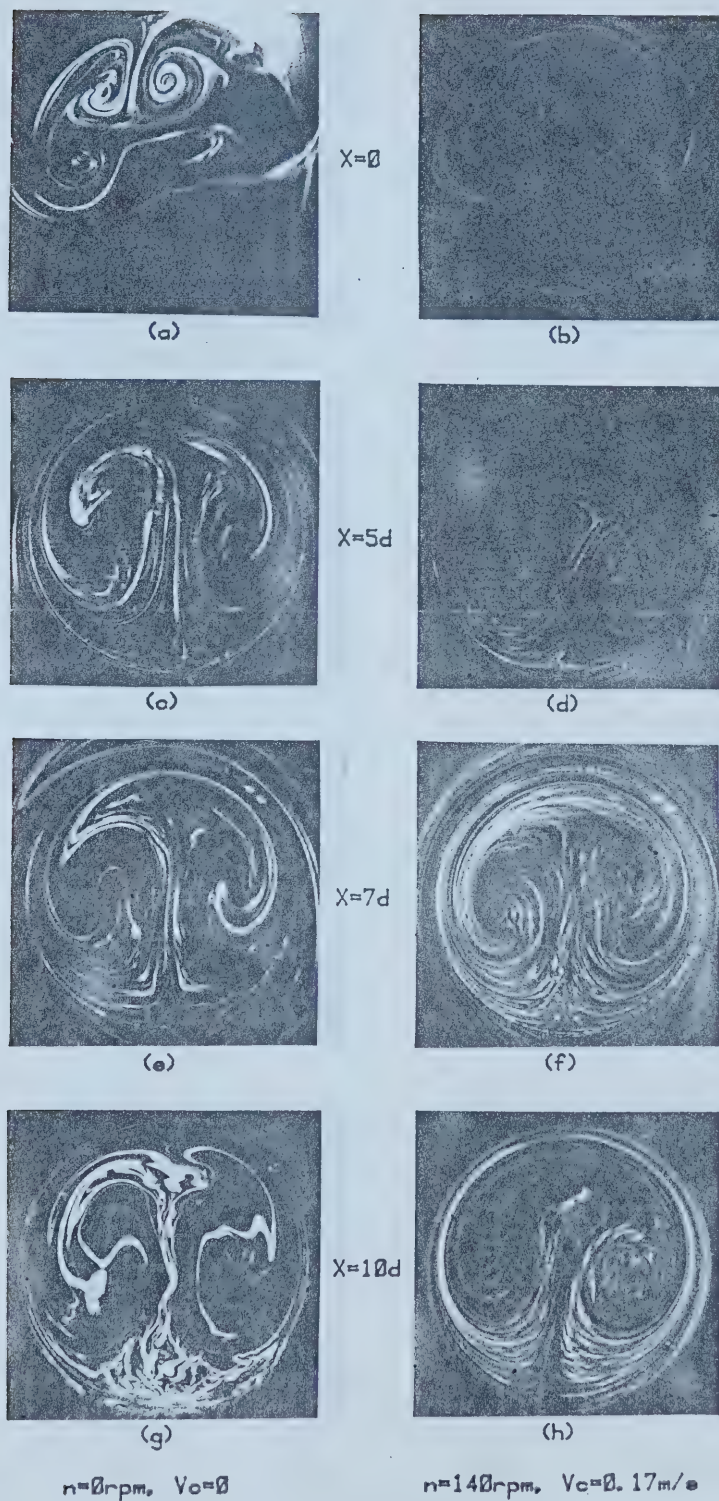


Fig. 8.5 Secondary flow patterns in a straight section downstream, $d=3.17\text{cm}$.

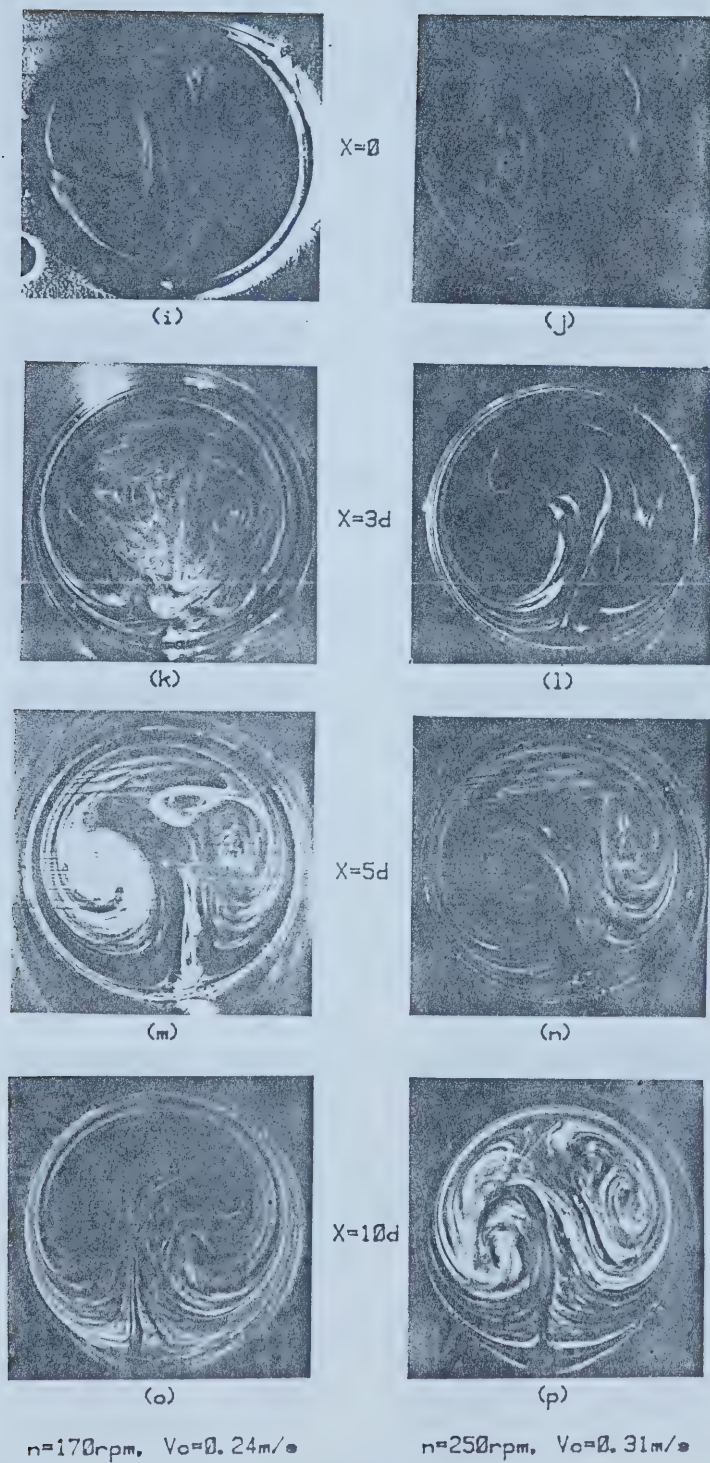


Fig. 8.5 Continued.

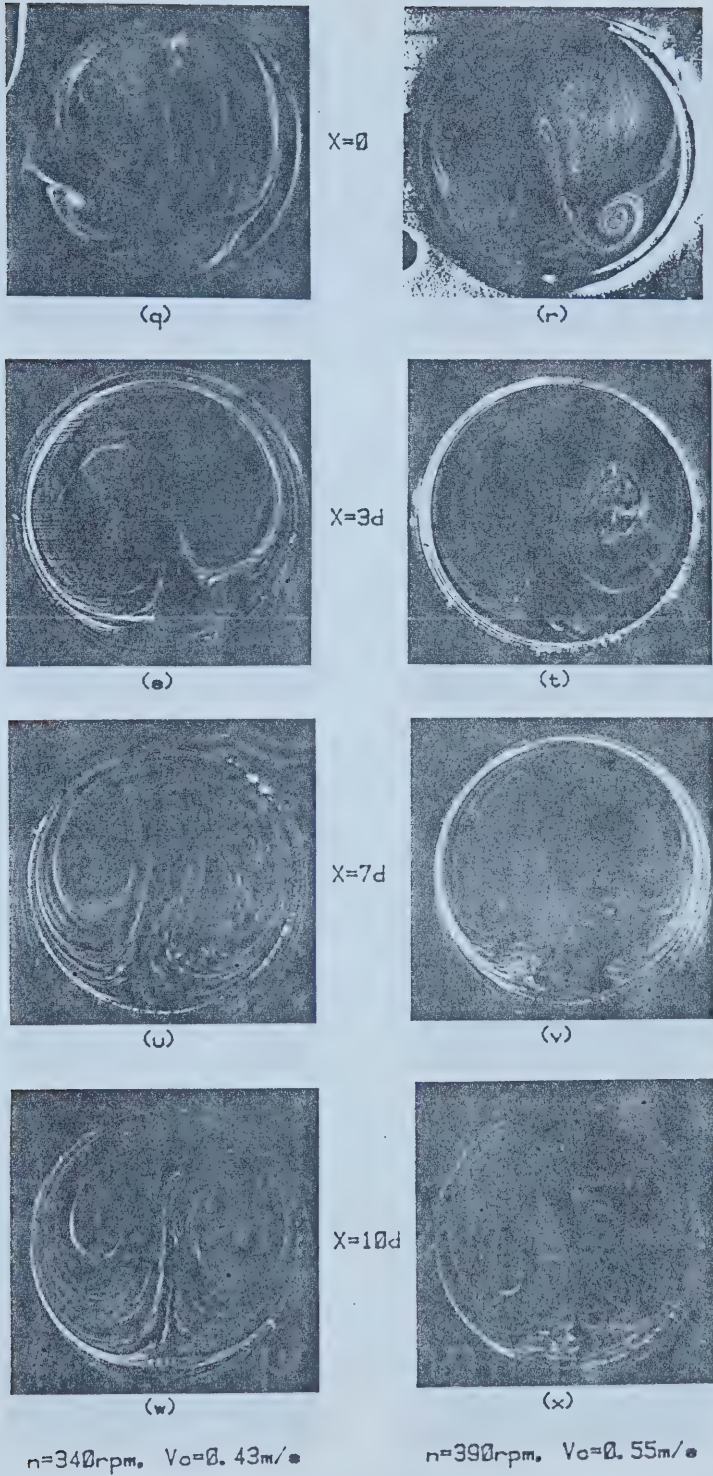


Fig. 8.5 Continued.

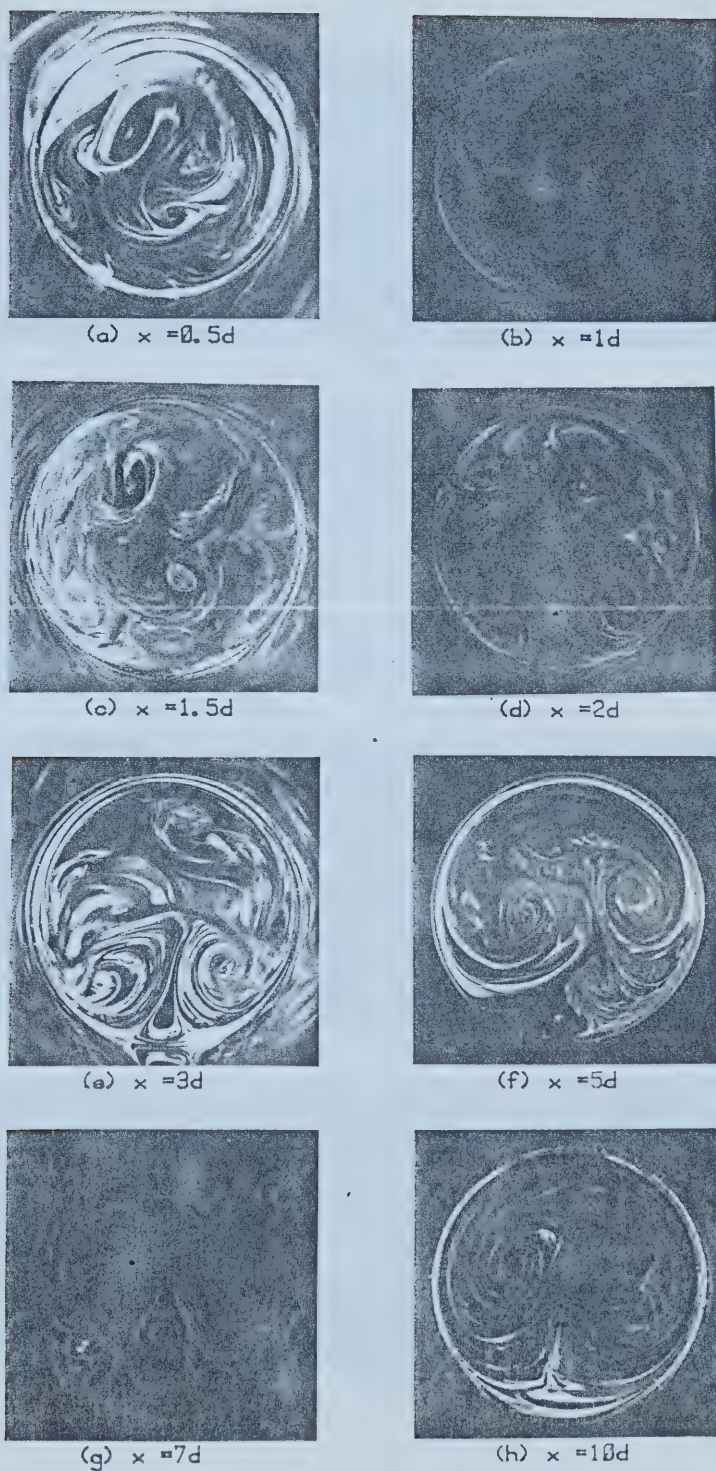
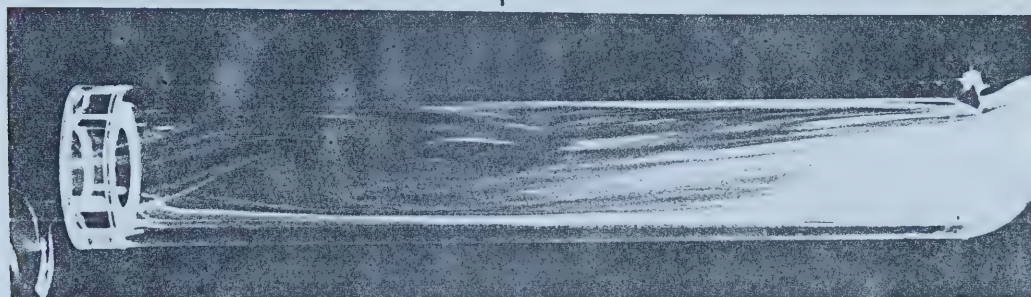


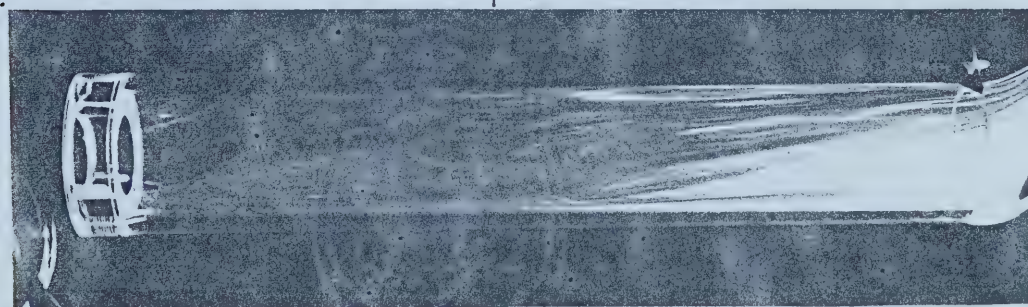
Fig. 8.6 Secondary flow patterns in a straight section downstream with abrupt change in diameter, $d=5.11\text{cm}$.



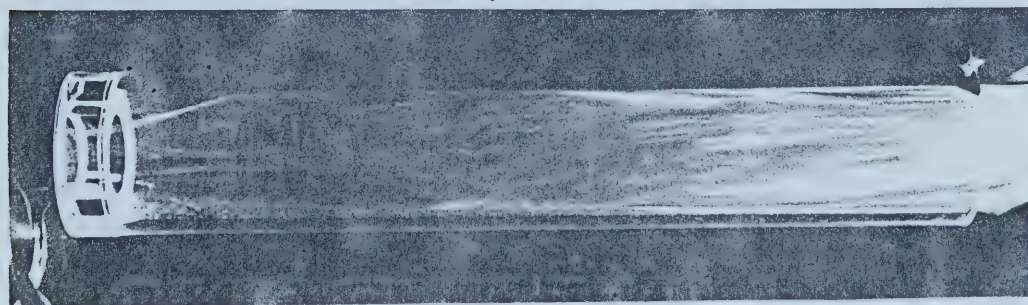
(a) $n=0\text{rpm}$, $V_c=0$



(b) $n=140\text{rpm}$, $V_c=0.09\text{m/s}$

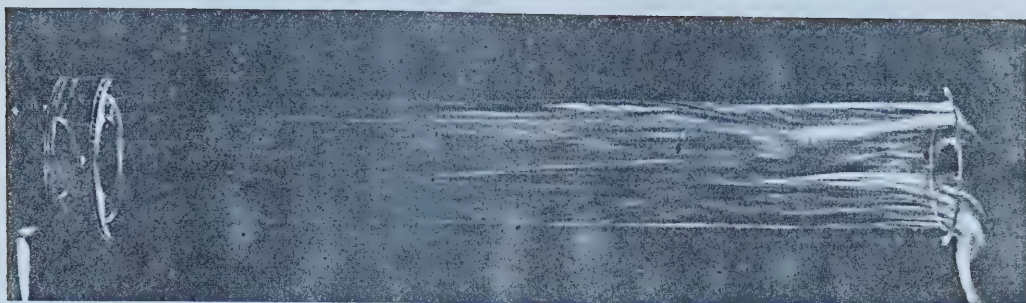


(c) $n=250\text{rpm}$, $V_c=0.15\text{m/s}$

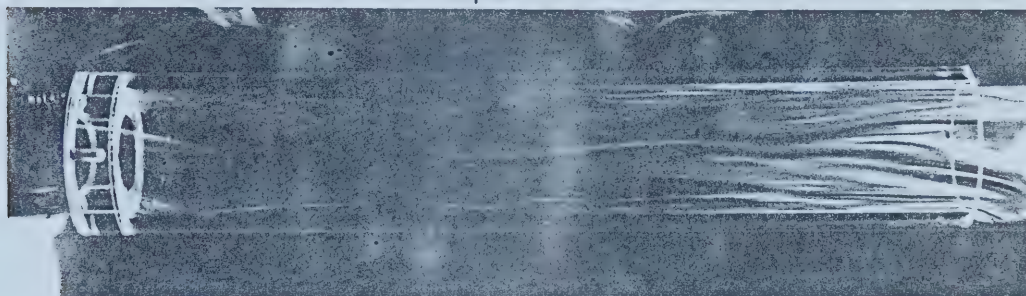


(d) $n=390\text{rpm}$, $V_c=0.30\text{m/s}$

Fig. 8.7 Side view of axial flow in the tube with $d=5.11\text{cm}$.



(a) $n=0\text{rpm}$, $V_c=0$



(b) $n=140\text{rpm}$, $V_c=0.09\text{m/s}$



(c) $n=170\text{rpm}$, $V_c=0.11\text{m/s}$



(d) $n=250\text{rpm}$, $V_c=0.15\text{m/s}$

Fig. 8.8 Plan view of axial flow in the tube with $d=5.11\text{cm}$.

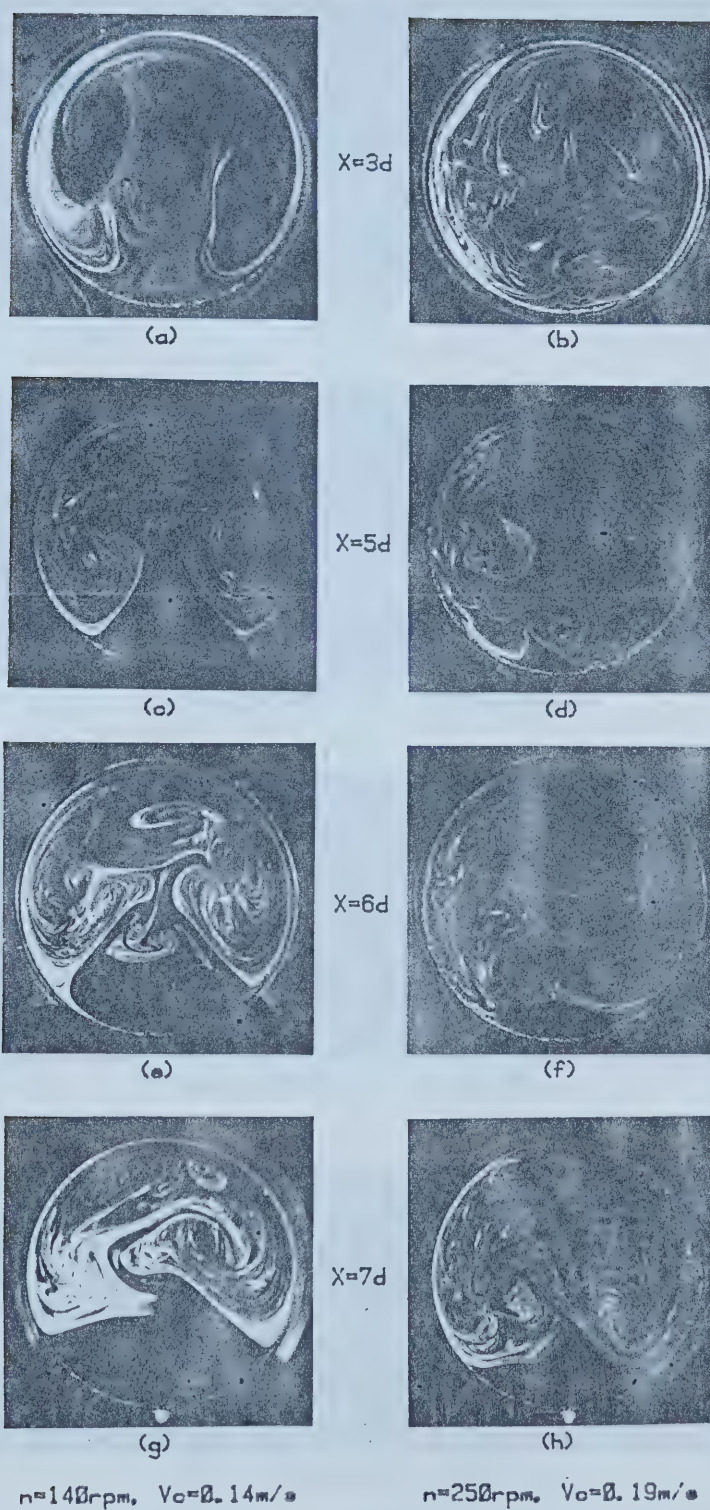


Fig. 8.9 Secondary flow patterns in a diffuser with $\beta=6.0^\circ$.



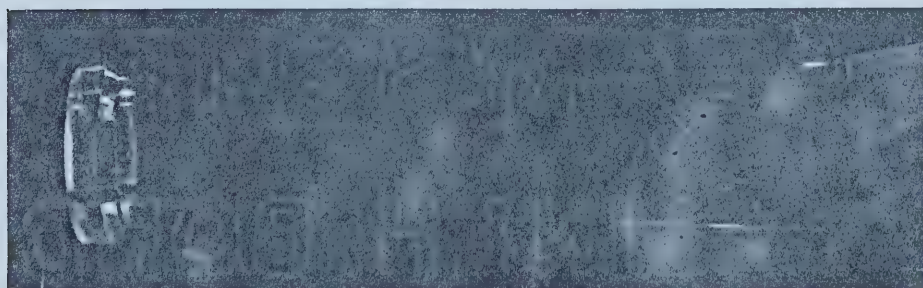
(a) $n=0\text{rpm}$, $V_c=0$



(b) $n=140\text{rpm}$, $V_c=0.14\text{m/s}$



(c) $n=390\text{rpm}$, $V_c=0.30\text{m/s}$



(d) $n=640\text{rpm}$, $V_c=0.42\text{m/s}$

Fig. 8.10 Side view of axial flow in a diffuser with $\beta=6.0^\circ$.



(a) $n=0\text{rpm}$, $V_c=0$



(b) $n=140\text{rpm}$, $V_c=0.14\text{m/s}$



(c) $n=250\text{rpm}$, $V_c=0.19\text{m/s}$



(d) $n=390\text{rpm}$, $V_c=0.30\text{m/s}$

Fig. 8.11 Plan view of axial flow in a diffuser with $\beta=6.0^\circ$.

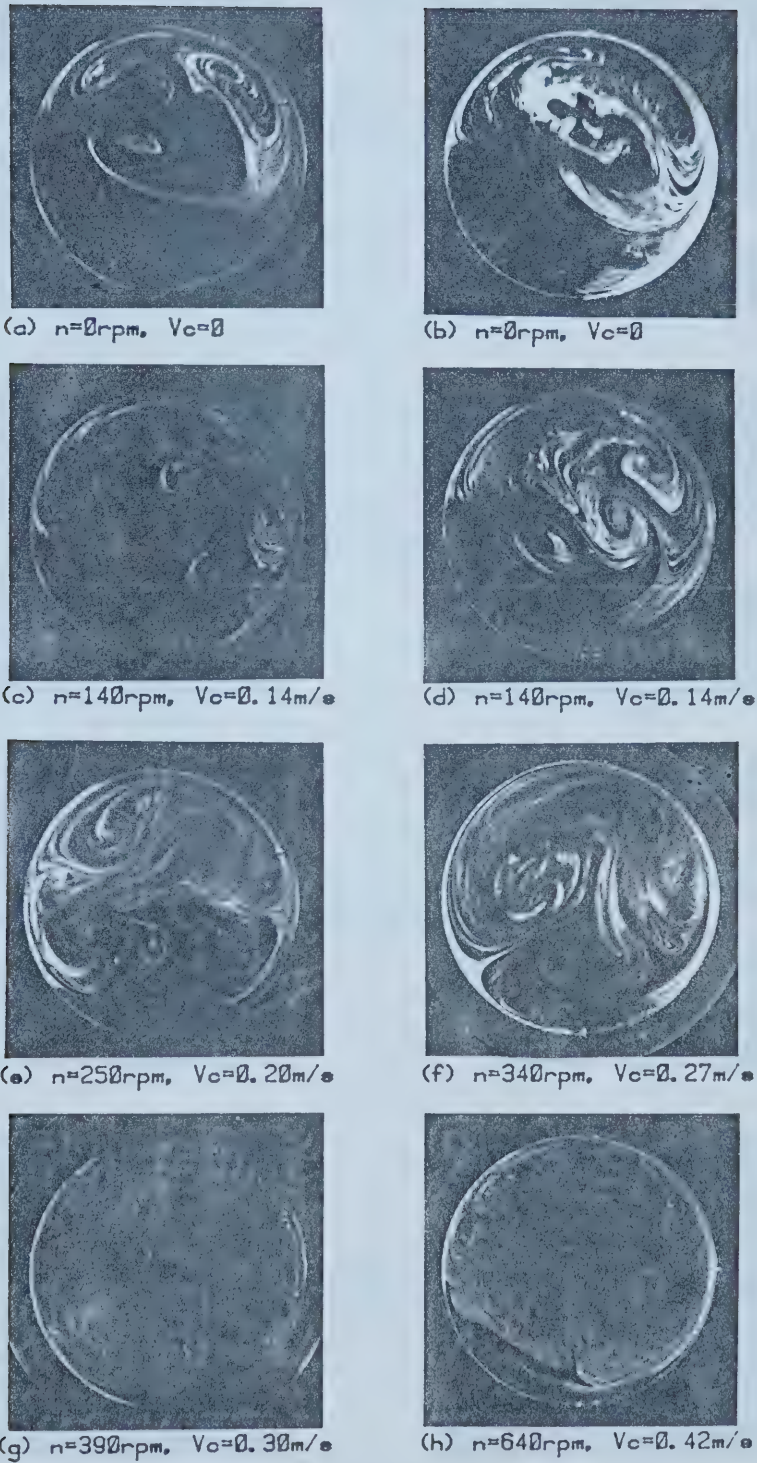
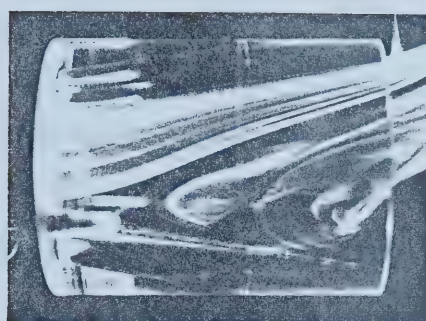
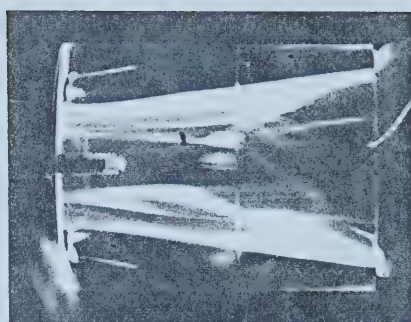


Fig. 8.12 Secondary flow patterns at the exits of two diffusers, left-hand side $\beta=14.1^\circ$, right-hand side $\beta=6.0^\circ$.



(a)

$n=0\text{rpm}$
 $V_c=0$



(b)

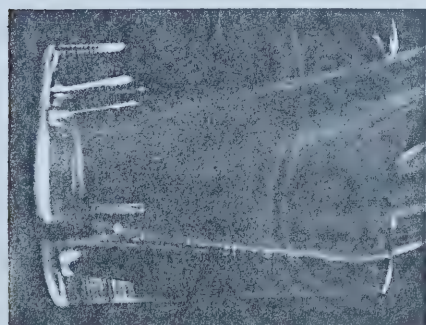


(c)

$n=140\text{rpm}$
 $V_c=0.14\text{m/s}$

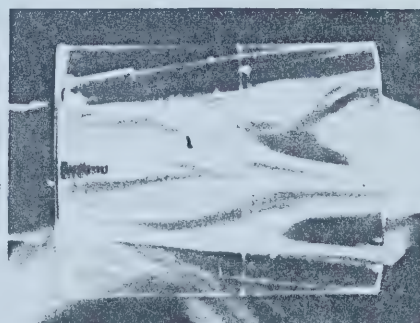


(d)

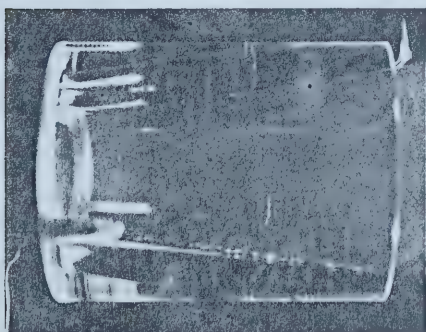


(e)

$n=170\text{rpm}$
 $V_c=0.17\text{m/s}$

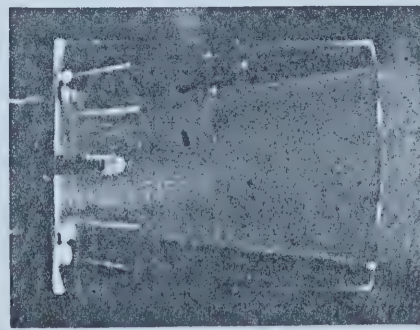


(f)



(g)

$n=390\text{rpm}$
 $V_c=0.30\text{m/s}$



(h)

Fig. 8.13 Side view (L.H.S.) and plan view (R.H.S.) of axial flow in a diffuser with $\beta=14.1^\circ$.

9. Conclusions

9.1 Scope of Results

Flow visualization has been done in this study to investigate the secondary flow patterns in unheated curved pipes, isothermally heated horizontal or inclined tubes, and isothermally heated curved pipes. A preliminary study has also been carried out to the secondary flow patterns at the exit of a centrifugal fan. The chosen flow visualization technique has demonstrated to be an effective method in obtaining the unique secondary flow patterns in curved or heated pipes. For most cases, the range of experimental parameters are so high that is often too difficult to be reached by numerical and analytical approaches.

A simple flow visualization experiment using the smoke injection method into air yields satisfactory and informative secondary flow patterns revealing the path of a field element on a cross-section of the pipe. Two different smoke generating techniques were employed in this thesis. The first one involved the injection of hydrochloric acid into air with ammonia vapor for the study of flow in unheated curved pipes. Since the location of hydrochloric acid injection is at the start of bend, the number of streamlines being seen is limited due to the short distance allowed for smoke to disperse into the flow field. The second smoke generating technique is by far the most satisfactory one; and was used for the rest of experiments.

Smoke is generated by burning paper straws or Chinese incense in a copper tube. Since the location of smoke injection is at the start of the set up, smoke has sufficient time to disperse into more secondary flow paths as the flow moves downstream. Thus, the streamlines exhibit more visual secondary flow patterns over the cross-section.

Perhaps the most important findings for these experimental studies are the realization of the complicated secondary flow patterns and the occurrence of a four-vortex solution in an isothermally heat curved pipe; and the investigation of the secondary flow development for the entry flow in curved pipes. Some minor findings include the confirmation of the existence of centrifugal instability in a curved circular pipe found by previous investigators; the observation of the inherent ascending developing secondary flow patterns in the thermal entrance region of isothermally heated horizontal and inclined tubes; and the investigation of the secondary flow patterns at the exit of a centrifugal fan.

In the presence of either centrifugal or buoyancy force effects, it is confirmed that the flow is still laminar even though the Reynolds number exceeds the critical value of 2300. The thermal entrance length for experiments in isothermally heated tubes is in the range of $z \geq 0.597$ which is considerably longer than $z = 0.05$ from the classical Graetz solution. As well, our results clearly demonstrated that the line of symmetry along the plane of curvature may not

necessary hold, especially at higher flow rate and in the transient state case.

These presented secondary flow patterns should be studied together with the related experimental and numerical results for the flow in pipes in the areas of the friction factor, pressure gradients, boundary layer, wall shear stresses, and velocity profiles. It is hoped the presented secondary flow patterns would provide some insight into the particular flow case and thus are useful for future comparison with predictions from numerical solutions.

9.2 Future Studies

In view of the satisfactory results obtained by this flow visualization technique, it is recommended that tubes of cross-sections other than circular be studied.

For the entry flow in curved tubes, secondary flow patterns subjected to the curvature effect have gone through a great change from the $\phi=0^\circ$ to $\phi=45^\circ$. It is suggested more work be done to clarify this situation.

For the thermal entrance region in isothermally heated pipes, the heat transfer coefficient can be readily evaluated once the mixed mean air temperature, instead of the centreline air temperature in our case, is accurately measured.

Our present smoke injection technique has opened up a wide range of possibilities for investigating secondary flow patterns in heated pipes. Finally, it is hoped that the

studies on secondary flow patterns in isothermally heated vertical pipes and in constant heat flux horizontal and inclined pipes be undertaken in the near future so as to complement the available numerical results.

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